8 Prolog (ACR)

(a) The propositional logic formula \( A \land B \) can be represented by the Prolog term `and(lit(A),lit(B))`.

Describe a scheme based on this example for representing an arbitrary propositional logic formula in Prolog. Demonstrate your scheme by encoding the formula \( \neg(\neg P \land (Q \lor \neg(R \land S))) \). [4 marks]

(b) A formula is in Conjunctive Normal Form (CNF) if it is expressed as a conjunction (\( \land \)-ing) of clauses, where each clause is a disjunction (\( \lor \)-ing) of literals.

Write a Prolog program for converting a propositional logic formula into CNF by implementing the following procedure:

(i) Push negations inwards until each applies only to a literal using De Morgan’s laws: \( \neg(A \lor B) \simeq \neg A \land \neg B \) and \( \neg(A \land B) \simeq \neg A \lor \neg B \) [5 marks]

(ii) Remove double negations of literals: \( \neg\neg A \simeq A \) [1 mark]

(iii) Distribute one disjunction from the formula over a conjunction or fail if no such disjunction exists: \( A \lor (B \land C) \simeq (A \lor B) \land (A \lor C) \) [6 marks]

(iv) Repeatedly apply the distribution step until no more distribution can be done [4 marks]

Ensure that your predicates behave appropriately with backtracking, avoid over-use of cut, and are commented appropriately. Minor syntactic errors will not be penalised.