5 Discrete Mathematics II (MPF)

(*a*) Let $\mathbb{N}_0$ be the set $\{0, 1, \cdots\}$ of natural numbers with zero and, for $k \in \mathbb{N}_0$, let $[k] = \{\, i \in \mathbb{N}_0 \mid i < k \,\}$.

For $m, n \in \mathbb{N}_0$:

(*i*) Define the disjoint union $[m] \uplus [n]$ of $[m]$ and $[n]$ together with a bijective function $[m] \uplus [n] \to [m + n]$. [4 marks]

(*ii*) Define the cartesian product $[m] \times [n]$ of $[m]$ and $[n]$ together with a bijective function $[m] \times [n] \to [m \cdot n]$. [4 marks]

(*iii*) Define the powerset $\mathcal{P}[m]$ of $[m]$ together with a bijective function $\mathcal{P}[m] \to [2^m]$. [4 marks]

(*iv*) Consider the set $\big([m] \Rightarrow [n]\big)$ of functions from $[m]$ to $[n]$ and define a bijective function $\big([m] \Rightarrow [n]\big) \to [n^m]$. [4 marks]

In each case, justify why the functions you have defined are bijective.

(*b*) For a set D, show that, if there exists a surjection from D to the set $(D \Rightarrow D)$ of functions from D to D, then D has exactly one element. You may use standard results provided you state them clearly.

[4 marks]