3 Discrete Mathematics I (SS)

(a) Consider the following assertions about the sets $A$, $B$ and $C$. Write them down in the language of predicate logic. Use only the constructions of predicate logic ($\forall$, $\exists$, $\neg$, $\Rightarrow$, $\land$, $\lor$) and the element-of symbol ($\in$). Do not use derived notions ($\cap$, $\cup$, $=$, etc.).

Example: “$A$ is a subset of $B$” can be formalized as $\forall x. x \in A \Rightarrow x \in B$.

(i) The sets $A$ and $B$ are equal.

(ii) Every element of $A$ is in the set $B$ or the set $C$.

(iii) If $A$ is disjoint from $B$ then $B$ and $C$ overlap.

[6 marks]

(b) State the principle of induction over lists. Use the language of predicate logic.

[2 marks]

(c) Consider the following functions over lists of integers, written in ML syntax.

```ml
fun app([],ys) = ys
  | app(x::xs,ys) = x::app(xs,ys);

fun rev([]) = []
  | rev(x::xs) = app(rev(xs),x::[]);

fun revapp([],ys) = ys
  | revapp(x::xs,ys) = revapp(xs,x::ys);
```

Prove that $\forall xs. \text{revapp}(xs,[]) = \text{rev}(xs)$

Your proof should be clear but it does not need to be a structured proof. You may use the abbreviation $xs @ ys$ for $\text{app}(xs,ys)$. You may assume the following facts.

$\forall xs. xs @ [] = xs$ $\forall xs,ys,zs. xs @ (ys @ zs) = (xs @ ys) @ zs$

Hint: first use induction to show that $\forall xs. \forall ys. \text{revapp}(xs,ys) = \text{app}(\text{rev}(xs),ys)$. [12 marks]