COMPUTER SCIENCE TRIPOS Part IA

Tuesday 4 June 2013 1.30 to 4.30

COMPUTER SCIENCE Paper 2

*Answer **one** question from each of Sections A, B and C, and **two** questions from Section D.*

*Submit the answers in five **separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
Approved calculator permitted

SECTION A

1 Digital Electronics

(a) A four-variable Boolean function is given by

$$F = A.B.C + B.C.D + A.\overline{C}.D$$

where $A.B.\overline{C}.\overline{D}$, $A.\overline{B}.C.D$ and $\overline{A}.\overline{B}.C.D$ are don't-care states. Using a Karnaugh map or otherwise:

(i) Find the simplest sum of products expression for F. [3 marks]

(ii) Design a circuit to implement F using NAND gates only. [3 marks]

(iii) Design a circuit to implement F using NOR gates only. [4 marks]

(b) Define static 1 and static 0 hazards. [4 marks]

(c) Consider the following multi-level Boolean function.

$$Y = A.B.C + (A + D).(\overline{A} + \overline{C})$$

(i) Determine with the aid of a Karnaugh Map, or otherwise, a minimised sum of products expression for Y that does not possess static 1 hazards. [3 marks]

(ii) Show with the aid of a Karnaugh map, or otherwise, that your proposed solution in part (c)(i) does not possess static 0 hazards. [3 marks]

2 Digital Electronics

(*a*) With the use of appropriate diagrams, briefly explain the operation of Moore and Mealy finite state machines, paying particular regard to their differences.

[6 marks]

(*b*) A serial data line carries binary data to a system with input X. The system is required to detect a sequence $0\,1\,0$ in the data and give an output $Y = 1$ at the end of the sequence. Only non-overlapping sequences should be detected in the data. For example, the output Y should only be 1 for the 0 underlined in the input sequence $\ldots\, 1\,0\,1\underline{0}\,1\,0\,1\,\ldots$.

Draw the state diagram for the system and state the minimum number of D-type bistables that are required to implement this finite state machine.

[6 marks]

(*c*) A finite state machine (FSM) has the following sequence of states at the outputs of its D-type state registers, Q_A , Q_B and Q_C :

Q_A	Q_B	Q_C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
0	0	0
⋮	⋮	⋮

if the state registers are initialised to state $0\,0\,0$, or

Q_A	Q_B	Q_C
1	1	1
1	1	0
1	0	1
1	1	1
⋮	⋮	⋮

if the state registers are initialised to state $1\,1\,1$.

Determine the next state combinational logic required to implement this FSM.

[8 marks]

SECTION B

3 Operating Systems

Consider a single processor system in which multiple processes are running.

(a) What does it mean for a process to be I/O bound? What does it mean for it to be CPU bound? [2 marks]

(b) What is the difference between preemptive and non-preemptive scheduling? Which one requires specific hardware support and what is that hardware support? [3 marks]

(c) Two processes, A and B, have the following sequential execution patterns:

A: [cpu 4 ms; I/O 2 ms; cpu 4 ms; I/O 2 ms; cpu 4 ms]
B: [cpu 1 ms; I/O 2 ms; cpu 1 ms; I/O 2 ms; cpu 1 ms]

I/O operations for the two processes do not interfere with each other and are blocking.

(i) If the processes are run consecutively one after another, what is the elapsed time for all to complete? [2 marks]

(ii) Sketch the execution pattern under non-preemptive scheduling and determine the total elapsed time for completion. You may assume that processes are scheduled in the order in which they become ready to run and that in the event of a tie A has priority over B. You may further assume that the scheduler and context switches take negligible time. [6 marks]

(iii) Repeat (c)(ii) but for a preemptive scheduler that operates on a time slice of 2 ms, that is, no process can run for more than 2 ms at a time (unless no other process is runnable). [6 marks]

(iv) Is there any evidence from your results for (c)(ii) and (c)(iii) of a significant advantage for either scheduling method? [1 mark]

4 Operating Systems

(*a*) What is the difference between a *logical* or *virtual* memory address and a *physical* memory address? [2 marks]

(*b*) Consider a variable which is bound to a single logical address for the duration of process execution. How is it possible for the variable to be bound to different physical addresses during process execution? [3 marks]

(*c*) In a demand paging system, what are the factors that can be taken into account when deciding which valid page should be made a victim? [5 marks]

(*d*) Consider a demand paging system in which hardware can trap unauthorised read or write accesses, but cannot perform any update to *use counts* or *written/dirty* bits. How can these be performed efficiently in software? [6 marks]

(*e*) What is *copy-on-write*? How can it be implemented in a demand paging system? [4 marks]

SECTION C

5 Discrete Mathematics II

(*a*) Let $\mathbb{N}_0$ be the set $\{0, 1, \cdots\}$ of natural numbers with zero and, for $k \in \mathbb{N}_0$, let $[k] = \{\, i \in \mathbb{N}_0 \mid i < k \,\}$.

For $m, n \in \mathbb{N}_0$:

(*i*) Define the disjoint union $[m] \uplus [n]$ of $[m]$ and $[n]$ together with a bijective function $[m] \uplus [n] \to [m + n]$. [4 marks]

(*ii*) Define the cartesian product $[m] \times [n]$ of $[m]$ and $[n]$ together with a bijective function $[m] \times [n] \to [m \cdot n]$. [4 marks]

(*iii*) Define the powerset $\mathcal{P}[m]$ of $[m]$ together with a bijective function $\mathcal{P}[m] \to [2^m]$. [4 marks]

(*iv*) Consider the set $\big([m] \Rightarrow [n]\big)$ of functions from $[m]$ to $[n]$ and define a bijective function $\big([m] \Rightarrow [n]\big) \to [n^m]$. [4 marks]

In each case, justify why the functions you have defined are bijective.

(*b*) For a set D, show that, if there exists a surjection from D to the set $(D \Rightarrow D)$ of functions from D to D, then D has exactly one element. You may use standard results provided you state them clearly.

[4 marks]

6 Discrete Mathematics II

Let $R \subseteq U \times U$ be a relation on a set U.

(a) Let $R^\dagger \subseteq U \times U$ be the relation inductively defined by the rules

$$\frac{}{(a,b)}\ (a,b) \in R \qquad\qquad \frac{(a,b) \quad (b,c)}{(a,c)}$$

and let $R^\bullet \subseteq U \times U$ be the relation inductively defined by the rules

$$\frac{}{(a,b)}\ (a,b) \in R \qquad\qquad \frac{(b,c)}{(a,c)}\ (a,b) \in R$$

Either prove or disprove the following statements.

(i) $R^\bullet \subseteq R^\dagger$ [4 marks]

(ii) $R^\dagger \subseteq R^\bullet$ [4 marks]

(b) Let $R^\diamond \subseteq U \times U$ be the relation inductively defined by the rules

$$\frac{}{(a,b)}\ (a,b) \in R \qquad\qquad \frac{(b,c)}{(a,d)}\ (a,b),(c,d) \in R$$

Either prove or disprove the following statements.

(i) $R^\diamond \subseteq \bigcup_{n \in \mathbb{N}_0} R^{2n+1}$ [4 marks]

(ii) $\bigcup_{n \in \mathbb{N}_0} R^{2n+1} \subseteq R^\diamond$ [4 marks]

(iii) $\left(R^\diamond\right)^{-1} = \left(R^{-1}\right)^\diamond$ [4 marks]

You may assume without proof that for each $n \in \mathbb{N}_0$, the relation $R^n \subseteq U \times U$ satisfies $R \circ R^n = R^{n+1} = R^n \circ R$ and $(R^n)^{-1} = (R^{-1})^n$.

SECTION D

7 Probability

Suppose that N is a random variable taking values $j = 0, 1, 2 \ldots$ with

$$\mathbb{P}(N = j) = (1 - \rho)\rho^j$$

and where $0 < \rho < 1$.

(a) Show that $\mathbb{P}(N = j)$ is a *probability mass function.* [2 marks]

(b) For $k = 0, 1, 2, \ldots$ derive an expression for $\mathbb{P}(N > k)$. [2 marks]

(c) Derive the *probability generating function*, $G_N(z)$, for the random variable N, stating carefully any conditions required for it to be well-defined and use it to determine

(i) $\mathbb{E}(N)$

(ii) $\mathrm{Var}(N)$

[8 marks]

(d) For a random variable X, which takes non-negative integer values, show that

$$\mathbb{E}(X) = \sum_{k=0}^{\infty} \mathbb{P}(X > k)\,.$$

[6 marks]

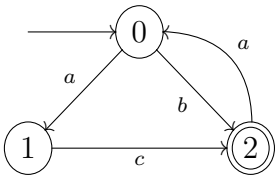
(e) By evaluating $\sum_{k=0}^{\infty} \mathbb{P}(N > k)$ show that this expression equals the value derived in part (c)(i) for $\mathbb{E}(N)$.

[2 marks]

8 Regular Languages and Finite Automata

(*a*) (*i*) Given any non-deterministic finite automaton M, describe how to construct a regular expression r whose language of matching strings $L(r)$ is equal to the language $L(M)$ accepted by M. [5 marks]

(*ii*) Give a regular expression r with $L(r) = L(M)$ when M is the following non-deterministic finite automaton.



[3 marks]

(*b*) State the Pumping Lemma and explain how it is used to prove that languages are not regular. [4 marks]

(*c*) Are the following languages regular? Justify your answer in each case.

(*i*) $L_1 = \{a^k b^m c^n \mid (k = m \text{ or } m = n) \text{ and } k + m + n \geq 2\}$

(*ii*) $L_2 = \{a^k b^m c^n \mid (k = m \text{ or } m = n) \text{ and } k + m + n \leq 2\}$

(*iii*) $L_3 = \{a^k b^m c^n \mid k + m + n \geq 2\}$ [8 marks]

9 Software and Interface Design

Imagine you want to design an online file-sharing system that works like Dropbox, but runs from your own house, using a Raspberry Pi with a large USB disk attached. You will use it to share files with your friends. You also hope that in future an open source version might lead to a larger number of servers for load balancing and shared resources.

(*a*) Describe in detail (at least 10 steps) a use case scenario where a system operator shares a file with someone who has not used the system before. [5 marks]

(*b*) Using an object-oriented language, identify at least four classes that will be involved in implementing this scenario. [2 marks]

(*c*) How would you test the performance of the system? [3 marks]

(*d*) How would you test the usability of the system? [2 marks]

(*e*) How might you identify additional system requirements for new users?

[2 marks]

(*f*) Consider how the system can be configured and modified. Identify three different types of person who might do this, and for each of them, describe how they would learn what to do.

[6 marks]

END OF PAPER