9 Optimising Compilers (AM)

(a) Explain the core ideas of strictness analysis, including the abstract values used for abstracting non-function values and what concrete values they represent. Briefly explain how program functions $f$ are abstracted to strictness functions $f^\sharp$. Give the abstractions of $\lambda(x,y). x + y$ and $\lambda(x,y). \text{if random()} \text{ then } x \text{ else } y$. [5 marks]

(b) Justify or correct the following statements: (i) “since abstract interpretation replaces real-world computation with a directly corresponding abstract computation then strictness analysis fails to terminate on non-terminating programs”; and (ii) “when a strict function is applied to an expression $e$ then $e$ is necessarily evaluated during the call”. [4 marks]

(c) We now wish to extend strictness analysis from simple $\text{int}$ expressions to allow also (lazy) $\text{int list}$ expressions. These represent lists whose head and tail components are only evaluated when required. Wadler suggested capturing strictness-like properties on lazy lists using an abstract interpretation with four abstract values for $\text{int list}$ concrete values:

- $0$: non-termination
- $\infty$: a chain of cons cells, either infinite or having some tail component which does not terminate
- $0\in$: a chain of cons cells ending in $\text{nil}$ but having at least one member which does not terminate
- $1\in$: a possibly empty chain of cons cells ending in $\text{nil}$ every member of which terminates

By analogy with ordinary strictness functions, give abstract interpretations in truth-table form (noting that values of type $\text{int list}$ have four values rather than the standard two) for the following functions involving lazy list values:

(i) $\lambda(x:\text{int list}). \text{nil}$ [1 mark]

(ii) $\lambda(x:\text{int list}). \text{cons}(42,x)$ [1 mark]

(iii) $\lambda(x,y:\text{int list}). \text{if random()} \text{ then } x \text{ else } y$

Explain how you resolved any choice which arose. [3 marks]

(iv) $\text{hd}$ [2 marks]

(v) $\text{tl}$ [1 mark]

(vi) $\text{append}$ [2 marks]

(vii) $\text{reverse}$ [1 mark]