COMPUTER SCIENCE TRIPOS Part II – 2012 – Paper 9

6 Information Theory and Coding (JGD)

(a) Prove that the information measure is additive: that the information gained from observing the combination of N independent events, whose probabilities are p_i for $i = 1 \ldots N$, is the *sum* of the information gained from observing each one of these events separately and in any order. [3 marks]

(b) What is the shortest possible decodable code length, in bits per average symbol, that could be achieved for a six-letter alphabet whose symbols have probabilities $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{32}\right)$? [3 marks]

(c) Consider Shannon's third theorem, the *Noisy Channel Coding Theorem,* for a continuous communication channel having bandwidth W Hertz, perturbed by additive white Gaussian noise of power spectral density N_0, and average transmitted power P.

(i) Is there any limit to the capacity of such a channel if you increase its signal-to-noise ratio $\dfrac{P}{N_0 W}$ without limit? If so, what is that limit? [4 marks]

(ii) Is there any limit to the capacity of such a channel if you can increase its bandwidth W in Hertz without limit, but while not changing N_0 or P? If so, what is that limit? [3 marks]

(d) For each of the four classes of signals in the left table below, identify its characteristic spectrum from the right table. ("Continuous" here means supported on the reals, *i.e.* at least piecewise continuous but not necessarily everywhere differentiable. "Periodic" means that under multiples of some finite shift the function remains unchanged.) Give your answer just in the form 1-A, 2-B, etc. Note that you have 24 different possibilities. [4 marks]

Class	*Signal Type*
1.	continuous, aperiodic
2.	continuous, periodic
3.	discrete, aperiodic
4.	discrete, periodic

Class	*Spectral Characteristic*
A.	continuous, aperiodic
B.	continuous, periodic
C.	discrete, aperiodic
D.	discrete, periodic

(e) Define the Kolmogorov algorithmic complexity K of a string of data. What relationship is to be expected between the Kolmogorov complexity K and the Shannon entropy H for a given set of data? Give a reasonable estimate of the Kolmogorov complexity K of a fractal, and explain why it is reasonable. [3 marks]