7 Hoare Logic (MJCG)

In this question we consider a semantics of \texttt{FOR}-commands in which
\[
\text{FOR } V := E_1 \text{ UNTIL } E_2 \text{ DO } C
\]
is defined to be equivalent to
\[
V := E_1; \text{ WHILE } V \leq E_2 \text{ DO } (C; \ V := V + 1)
\]

(a) How does this semantics of \texttt{FOR}-commands differ from the one given in the lectures? \[4 \text{ marks}\]

(b) The following \texttt{FOR}-rule is similar to one proposed by John Wickerson:
\[
\begin{align*}
\vdash P & \Rightarrow R[E_1/V] & \vdash R \land V > E_2 & \Rightarrow Q & \vdash \{R \land V \leq E_2\} C \{R[V+1/V]\}
\end{align*}
\]

Assuming the semantics of \texttt{FOR}-commands given above, derive this Wickerson-style \texttt{FOR}-rule from the standard axioms and rules of Hoare logic. \[10 \text{ marks}\]

(c) Is the \texttt{FOR}-axiom:
\[
\vdash \{P \land E_2 < E_1\} \text{ FOR } V := E_1 \text{ UNTIL } E_2 \text{ DO } C \{P\}
\]
sound with the semantics given above? Justify your answer either with a proof of this axiom, or with a counterexample. \[6 \text{ marks}\]