5 Logic and Proof (LCP)

(a) Exhibit a model for the following set of formulas, or prove that none exists.

\[ P \rightarrow Q \land R \quad P \land Q \rightarrow S \quad \neg R \leftrightarrow S \quad P \lor Q \]

[8 marks]

(b) Consider the following set of clauses:

\{\neg(x < y), -y < -x\} \quad \{\neg(x < y), x + z < y + z\} \quad \{0 < 1\}

(i) What is the Herbrand universe of these clauses? [3 marks]

(ii) What semantics must any Herbrand interpretation of the clauses attach to the function symbols? [3 marks]

(iii) Specify an Herbrand model by giving a semantics of the relation $<$, justifying your choice with reference to a natural model of the set of clauses. [6 marks]