1 Complexity Theory (AD)

(a) Suppose $L_1$ and $L_2$ are languages in $\mathbf{P}$. What can you say about the complexity of each of the following? Justify your answer in each case.

(i) $L_1 \cup L_2$. [3 marks]

(ii) $L_1 \cap L_2$. [3 marks]

(iii) The complement of $L_1$. [2 marks]

(b) Suppose $L_1$ and $L_2$ are languages in $\mathbf{NP}$. What can you say about the complexity of each of the following? Justify your answer in each case.

(i) $L_1 \cup L_2$. [3 marks]

(ii) $L_1 \cap L_2$. [3 marks]

(iii) The complement of $L_1$. [2 marks]

(c) Give an example of a language in $\mathbf{NP}$ that is not $\mathbf{NP}$-complete and prove that it is not. [4 marks]