COMPUTER SCIENCE TRIPOS Part IA

Tuesday 5 June 2012 1.30 to 4.30

COMPUTER SCIENCE Paper 2

Answer ***one*** *question from each of Sections A, B and C, and* ***two*** *questions from Section D.*

Submit the answers in five ***separate*** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* ***all*** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Graph paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
Approved calculator permitted

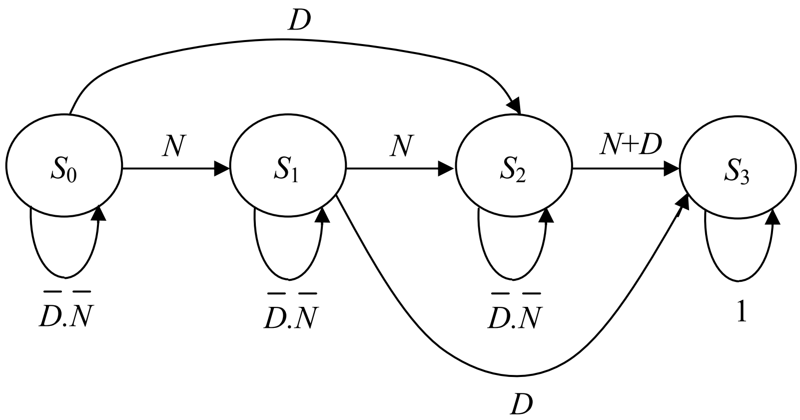
SECTION A

1 Digital Electronics

(*a*) Show how a transparent D latch can be constructed using an RS latch and some combinational logic gates. Briefly describe the operation of such a transparent D latch. [4 marks]

(*b*) A 3-bit synchronous counter has a mode control input X. If $X = 0$, the counter steps through the binary sequence 111, 110, 101, 100, 011, 010, 001, 000, and repeat, or if $X = 1$, the counter advances through the Grey code sequence 111, 101, 100, 000, 001, 011, 010, 110, and repeat. Draw the state diagram for the counter. [6 marks]

(*c*) A machine has the state diagram shown below, where N and D are two inputs and $N = D = 1$ cannot occur. The state assignment is $S_0 = [00]$, $S_1 = [01]$, $S_2 = [10]$ and $S_3 = [11]$, where the machine starts in state S_0 and finishes in state S_3. Note that state $= [Q_1Q_0]$ where Q_n is the output of flip-flop n.



(*i*) Write down the state transition table for this machine. [6 marks]

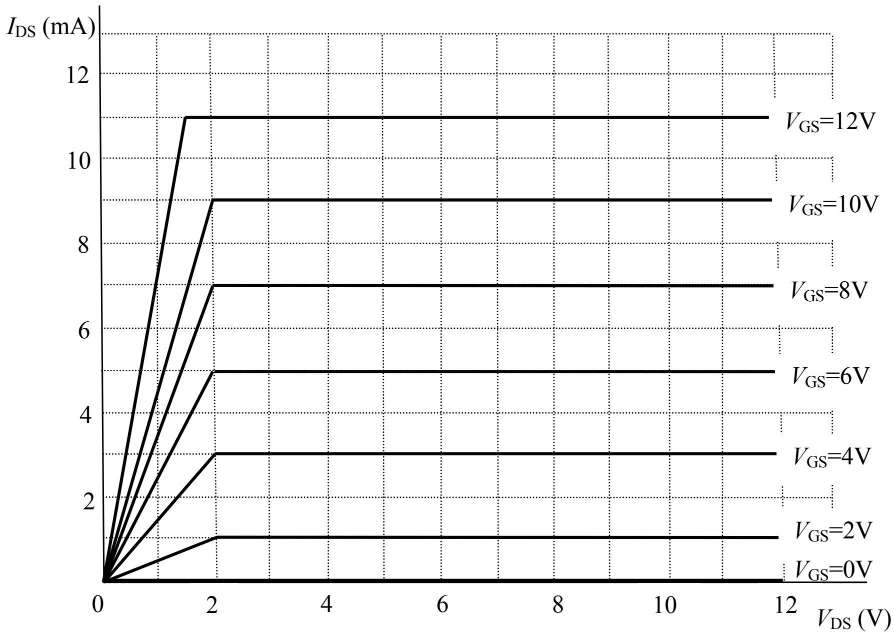
(*ii*) Assuming the use of D-type flip-flops for the state registers, determine the minimised Boolean expressions for the next state functions. [4 marks]

2 Digital Electronics

(*a*) With the aid of appropriate diagrams, show how the Source–Drain current that flows in a p-channel MOSFET is controlled by the applied Gate–Source voltage. [4 marks]

(*b*) (*i*) Draw the circuit diagram of a NOT gate that comprises an n-channel MOSFET and a resistor R. [2 marks]

(*ii*) For the NOT gate in (*b*)(*i*), plot the relationship between the input voltage, V_{in} and the output voltage, V_{out}. Assume that the power supply voltage $V_{\text{DD}} = 12$ V, $R = 1$ kΩ, and that the MOSFET has the characteristics given in the following figure. [4 marks]



(*c*) For the NOT gate in (*b*), calculate the power dissipated by the entire gate and that by resistor R alone, when $V_{\text{in}} = 12$ V. [4 marks]

(*d*) The stray capacitance present at the output of the NOT gate in (*b*) can be represented by a capacitor, $C = 100$ nF connected between the gate output and 0 V. Also assume that the MOSFET has an ON resistance $R_{\text{on}} = 100\ \Omega$. The input signal, V_{in}, is a 1 kHz square wave with minimum and maximum amplitudes of 0 V and 12 V respectively.

(*i*) Sketch the output signal waveform, V_{out}, of the NOT gate being sure to include indicative rise and fall times and voltage levels. [4 marks]

(*ii*) How could the rise-time of V_{out} be reduced and what would be the impact of your proposed solution on the power dissipation of the circuit? [2 marks]

SECTION B

3 Operating Systems

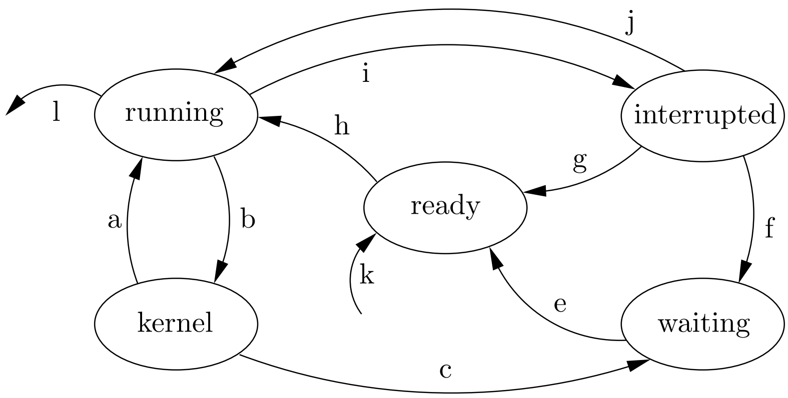
(*a*) To switch between processes, an operating system must save the context of the current executing process and restore the context of that being resumed.

(*i*) By what mechanism is the point of execution of a process preserved and restored? [2 marks]

(*ii*) Describe two methods by which the contents of a process address space are preserved and restored. [8 marks]

(*iii*) Give two other elements of process context. [2 marks]

(*b*) The diagram below is a simplified state transition diagram for a process in a generic operating system.



For each of the following, describe the transitions taken by the process and say whether or not it is necessary for the process scheduler to run:

(*i*) A running process is interrupted by a timer. The timer interrupt service routine determines that the time slice given to the process has not yet expired. [2 marks]

(*ii*) As in (*b*)(*i*) but the time slice given to the process has expired. [2 marks]

(*iii*) A running process makes a system call which can be serviced immediately by the kernel. [2 marks]

(*iv*) A running process makes a system call which requires an I/O operation to be initiated. [2 marks]

4 Operating Systems

Consider the following scheme for structuring a file from a set of disk blocks. A disk block contains 4096 bytes and a block address is 32 bits. The first block of the file contains the following information:

control information:	1024 bytes
direct block pointers:	1024 bytes
indirect block pointer:	4 bytes
double indirect block pointer:	4 bytes
immediate data:	2040 bytes

The data bytes of the file start at the beginning of the immediate data. After the immediate data, the file data is found on the block addressed by the first direct block pointer and then carries on in a fashion similar to the structure defined by a Unix inode. We consider the first byte of the file to be byte 0, then byte 1, etc.

(*a*) For each of the following describe the actions taken to fetch the indicated byte of a file, and state how many disk blocks may need to be read:

(*i*) byte 70 of the file [1 mark]

(*ii*) byte $2^{20} + 2044$ [1 mark]

(*b*) How large can a file be if it is to be guaranteed that only three disk blocks need to be read in order to access any given byte of the file? [4 marks]

(*c*) Information about a file can be stored in a directory that references the file or in the control part of the first block of the file (i.e. inode in Unix). Which of these is used in the Unix file system to store the following information and why?

(*i*) time of creation [3 marks]

(*ii*) file name [3 marks]

(*iii*) file access rights [3 marks]

(*d*) Another way to structure files on a disk is to use physically contiguous blocks (with contiguous addresses), so that if the first block of a file is block b, then the next block of the file is $b + 1$. Suppose we use this method, retain the control information on the first block, but include the first 3 KBytes of the file in the first block. Comment on the performance of such a system, considering reading, writing, and creating files. [5 marks]

SECTION C

5 Discrete Mathematics II

Let A, B, C be sets with relations $R \subseteq A \times B$ and $S \subseteq B \times C$.

(*a*) Give the definition of the relational composition $S \circ R$. [1 mark]

(*b*) The functions p, q, r are defined as follows:

$$\begin{aligned} p &: A \times B \times C \to A \times B \quad \text{such that} \quad p(a,b,c) = (a,b) \\ q &: A \times B \times C \to B \times C \quad \text{such that} \quad q(a,b,c) = (b,c) \\ r &: A \times B \times C \to A \times C \quad \text{such that} \quad r(a,b,c) = (a,c) \end{aligned}$$

(*i*) Describe the inverse images $p^{-1}R$ and $q^{-1}S$. [2 marks]

(*ii*) For $X \subseteq A \times B \times C$ describe its direct image $r\,X$ under r. [2 marks]

(*iii*) Prove that the relational composition $S \circ R$ equals $r\,(p^{-1}R \cap q^{-1}S)$, the direct image of the set $p^{-1}R \cap q^{-1}S$ under r. [5 marks]

(*c*) Suppose the relations R and S are countable. Is the relational composition $S \circ R$ countable? Justify your answer. [*Note:* You may use any well-known results provided you state them clearly.] [6 marks]

(*d*) Suppose now that $A = B = C$ and that both R and S are well-founded relations. Is the relational composition $S \circ R$ well-founded? Justify your answer. [4 marks]

6 Discrete Mathematics II

(*a*) State the principle of rule induction. [2 marks]

(*b*) Let X be the smallest subset of $\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$ such that

$2 \in X$ and $6 \in X$, and
if $x \in X$ and $y \in X$ then their product $x \times y \in X$.

(*i*) Using rule induction show $X \subseteq \{2^m 3^n \mid m, n \in \mathbb{N}_0\}$. [4 marks]

(*ii*) Is $18 \in X$? Justify your claim. [4 marks]

(*iii*) Describe a property $Q(m, n)$, where $m, n \in \mathbb{N}_0$, such that

$$X = \{2^m 3^n \mid m, n \in \mathbb{N}_0 \ \& \ Q(m, n)\}$$

[5 marks]

(*iv*) Prove your claim for part (*b*)(*iii*). [5 marks]

SECTION D

7 Probability

(*a*) A biased coin has probability p, $0 < p < 1$, of showing heads on a single throw. Show that the probability generating function of the random variable, X, giving the number of heads in n independent throws, is given by

$$G_X(z) = (pz + 1 - p)^n$$

[4 marks]

(*b*) Now suppose that the coin is thrown N times where N is a random variable with $\mathbb{E}(N) = \mu_N$ and $\mathrm{Var}(N) = \sigma_N^2$ and let Y be the random number of heads obtained.

(*i*) Show that

$$G_Y(z) = G_N(pz + 1 - p)$$

where $G_N(z)$ is the probability generating function of N. [4 marks]

(*ii*) Find $\mathbb{E}(Y)$ and $\mathrm{Var}(Y)$. [4 marks]

(*c*) Suppose that N has a Poisson distribution with parameter $\lambda > 0$.

(*i*) Find $G_N(z)$. [4 marks]

(*ii*) Show that Y has a Poisson distribution with parameter λp. [4 marks]

8 Regular Languages and Finite Automata

If r and s are regular expressions, write $r \preceq s$ to mean that the language of strings matching r is contained in the language of strings matching s.

(*a*) Show that if $r_1 \preceq s_1$ and $r_2 \preceq s_2$, then $r_1 r_2 \preceq s_1 s_2$. [2 marks]

(*b*) Show that if $r \preceq s$, then $r^* \preceq s^*$. [2 marks]

(*c*) Suppose $s \preceq t$ and $rt \preceq t$. Prove by induction that $r^n s \preceq t$ holds for all $n \geq 0$; deduce that $r^* s \preceq t$. [3 marks]

(*d*) Which of the following instances of the $\preceq$ relation are valid? In each case either give a proof, or specific examples of r and s for which the relation fails to hold. [*Hint:* You may find part (*c*) helpful for some of the proofs.]

(*i*) $r^* \mid s^* \preceq (r \mid s)^*$ [1 mark]

(*ii*) $(r \mid s)^* \preceq r^* \mid s^*$ [1 mark]

(*iii*) $(r^* s^*)^* \preceq (r \mid s)^*$ [2 marks]

(*iv*) $(r \mid s)^* \preceq (r^* s^*)^*$ [2 marks]

(*v*) $(rs \mid r)^* r \preceq r(sr \mid r)^*$ [2 marks]

(*e*) Briefly explain why there exists an algorithm for deciding whether or not $r \preceq s$ holds for any given regular expressions r and s (over some fixed alphabet). [5 marks]

9 Software Design

Consider a restaurant software ordering system. The system should allow the waiter to handle customers' orders by adding them, cancelling them, scheduling parts of the order (to make the starter arrive earlier than the main course). The cashier should be allowed to let customers pay for their meals, and print receipts.

(*a*) List two requirements (one functional and one non-functional) of the system using the MOSCOW requirement prioritisation method. [2 marks]

(*b*) Draw a use case diagram for a restaurant ordering system and give the use case detailed description of the use case CancelOrder. [7 marks]

(*c*) Define a class diagram with a maximum of three classes for the restaurant ordering system. [2 marks]

(*d*) Show the activity of processing a meal order within the system using an activity diagram. [4 marks]

(*e*) Show the use case realization (through interaction diagrams) of one of the use cases you have drawn for part (*b*). [5 marks]

END OF PAPER