2011 Paper 9 Question 4

Digital Signal Processing

(*a*) Make the following statements correct by changing one word or number. (Negating the sentence is not sufficient.)

  (*i*) A square-summable sequence is also called power signal. [1 mark]

  (*ii*) Adding together sine waves of the same frequency always results in another sine wave of the same phase. [1 mark]

  (*iii*) The Fourier transform of a Dirac comb is a Dirac impulse. [1 mark]

  (*iv*) 60 dBm = 1 W [1 mark]

(*b*) Before resampling a digital image at a quarter of its original resolution, you want to apply an anti-aliasing low-pass filter.

  (*i*) If you apply a 1-dimensional filter with impulse response $\{h_n\}$ both horizontally and vertically to image pixels $I_{x,y}$, what are the resulting filtered pixel values $\tilde{I}_{x,y}$? [4 marks]

  (*ii*) What would be the discrete impulse response $\{h_n\}$ of an ideal low-pass filter for this application, if its length were of no concern? [4 marks]

  (*iii*) You decide to truncate the impulse response $\{h_n\}$ at its second zero-crossing on each side, resulting in a new impulse response $\{\bar{h}_n\}$. In the frequency domain, this results in $\bar{H} = H * T$ for what function T? [4 marks]

  (*iv*) In order to make the frequency-domain response of your filter $\{\bar{h}_n\}$ smoother, you convolve it in the frequency domain with a rectangular pulse, the width of which is twice the distance between the zero crossings of T. What does the resulting time-domain impulse response $\{\breve{h}_n\}$ look like? [4 marks]