2011 Paper 9 Question 13

Topics in Concurrency

This question concerns CTL$^-$, a variant of CTL in which assertions are of the form:

$$A := T \mid A_0 \vee A_1 \mid A_0 \wedge A_1 \mid \neg A \mid \langle \lambda \rangle\, A \mid \langle - \rangle\, A \mid \mathbf{AG}\, A \mid \mathbf{EG}\, A$$

where λ ranges over action names. Recall that a *path* π from state s is a maximal sequence of states $\pi = (\pi_0, \pi_1, \pi_2, \ldots)$ such that $s = \pi_0$ and $\pi_i \overset{\cdot}{\longrightarrow} \pi_{i+1}$ for all i. The logical connectives have the standard interpretation and T represents "true". The interpretation of the modalities is:

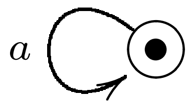
$s \models \langle \lambda \rangle\, A$	iff	there exists s' such that $s \xrightarrow{\lambda} s'$ and $s' \models A$
$s \models \langle - \rangle\, A$	iff	there exists s' such that $s \overset{\cdot}{\longrightarrow} s'$ and $s' \models A$
$s \models \mathbf{EG}\, A$	iff	for some path π from s, we have $\pi_i \models A$ for all i
$s \models \mathbf{AG}\, A$	iff	for all paths π from s, we have $\pi_i \models A$ for all i

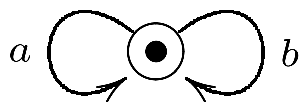
(a) What is the interpretation of the CTL modality $\mathbf{E}[A_0\ \mathbf{U}\ A_1]$? How can it be used to express the CTL$^-$ modality $\mathbf{AG}\, A$? [4 marks]

(b) Consider the following three formulae:

$$A_1 : \ \ \mathbf{AG}\, \langle a \rangle\, T \qquad A_2 : \ \ \neg\, \mathbf{AG} \neg\, \langle b \rangle\, T \qquad A_3 : \ \ \neg\, \mathbf{EG}\, \langle b \rangle\, T$$

(i) For each of the following two transition systems, state which of A_1, A_2 and A_3 are satisfied in the initial state.

P_1 : 

P_2 : 

[4 marks]

(ii) Draw a transition system with an initial state that satisfies $A_1 \wedge A_2 \wedge A_3$. [3 marks]

(c) Give a modal-μ formula that corresponds to the CTL$^-$ formula $\mathbf{AG}\, \langle a \rangle\, T$. [2 marks]

(d) Let the function φ on sets of states of a transition system be defined as:

$$\varphi(X) \overset{\text{def}}{=} \langle a \rangle\, T \vee \langle - \rangle\, X$$

Show by induction on $n \geq 1$ that

$$s \models \varphi^n(\emptyset) \quad \text{iff} \quad \begin{array}{l}\text{there exists } m \leq n \text{ and states } s_1, \ldots, s_m, s' \\ \text{such that } s = s_1 \overset{\cdot}{\longrightarrow} \ldots \overset{\cdot}{\longrightarrow} s_m \xrightarrow{a} s'\end{array}$$

Deduce that $s \models \mu X.\, \langle A \rangle\, T \vee \langle - \rangle\, X$ in a finite-state transition system if, and only if, $s \models \neg\, \mathbf{AG} \neg\, \langle a \rangle\, T$. [7 marks]