2011 Paper 8 Question 10

Principles of Communications

(*a*) Draw the transition rate diagram for an $M/M/1$ queueing system, explaining each type of label, symbol and transition you have used. [5 marks]

(*b*) Suppose an $M/M/1$ server is being designed with the following business in mind:

Each customer arrival earns the service 5 Euros. However, for each unit of time the customer waits in the system, there is a refund of 1 Euro.

(*i*) What is the range of arrival rates for which the system makes a net profit? [10 marks]

[Hint: You may wish to use the result that $P_j = \rho^j(1-\rho)$, and (hence) $P_0 = 1-\rho$, where P_i is the probability of i customers being in the service and $\rho = \lambda/\mu$ is the *utilisation* of the service, for the mean arrival rate of a Poisson process, λ and a mean service time μ.]

(*ii*) Imagine that a priority queue is made available for an additional fee. Discuss qualitatively the relationship between the customers' willingness to pay, and the appropriate setting of the fee to continue to maximise profit. [5 marks]