Logic and Proof

(a) Recently, automated theorem provers based on the saturation algorithm have become very powerful tools.

(i) Exhibit a proof by resolution of the following formula in first-order logic. Include the conversion into a set of clauses and provide brief justification for each step of the proof.

\[ \forall x (P(x) \rightarrow Q(x)) \rightarrow (\exists y P(y) \rightarrow \exists z Q(z)) \]

[6 marks]

(ii) Prove \( P(s(s(s(0)))) \) by linear resolution from the following assumptions:

\[
\forall x((P(x) \land Q(x)) \rightarrow P(s(x)))
\]

\[
\forall x(P(x) \rightarrow Q(x))
\]

\[ P(0) \]

[7 marks]

(b) Binary decision diagrams (BDDs) can be used to represent formulae in propositional logic.

Show the steps in the recursive construction of a BDD, ordered alphabetically, for the following formula:

\[ ((P \land Q) \lor R) \rightarrow (Q \lor R) \]

[7 marks]