

## 2011 Paper 6 Question 10

### Semantics of Programming Languages

The following grammar specifies the types and expressions of a simple functional programming language.

$$\begin{array}{l} \text{Types:} \quad T ::= \mathbf{int} \mid T \rightarrow T' \\ \text{Expressions:} \quad e ::= n \mid x \mid e + e' \mid \mathbf{fn}(x : T) \Rightarrow e \mid e e' \end{array}$$

where  $n$  ranges over all integers, and  $x$  ranges over variables.

- (a) Give a reasonable semantics for this language, by specifying a type system and a reduction relation. Use the call-by-name evaluation order. [9 marks]
- (b) Write down all the reduction steps of the following expression. You do not need to give their derivations.

$$(\mathbf{fn}(x : \mathbf{int}) \Rightarrow (\mathbf{fn}(x : \mathbf{int}) \Rightarrow x + x))(1 + 2)(3 + 4)$$

[3 marks]

- (c) Prove the following property of substitution. [Hint: Use rule induction for  $\Gamma, x : T \vdash e' : T'$ .]

$$\text{if } \Gamma \vdash e : T \text{ and } \Gamma, x : T \vdash e' : T' \text{ with } x \notin \text{dom}(\Gamma) \text{ then } \Gamma \vdash \{e/x\}e' : T'$$

[8 marks]