Regular Languages and Finite Automata

(a) Give a regular expression \( r \) over the alphabet \( \Sigma = \{a, b, c\} \) such that the language determined by \( r \) consists of all strings that contain at least one occurrence of each symbol in \( \Sigma \). Briefly explain your answer. [5 marks]

(b) Let \( L \) be the language accepted by the following non-deterministic finite automaton with \( \varepsilon \)-transitions:

(i) Draw a deterministic finite automaton that accepts \( L \).

(ii) Write down a regular expression that determines \( L \).

Briefly explain your answers. [5 marks]

(c) Show that if a deterministic finite automaton \( M \) accepts any string at all, then it accepts one whose length is less than the number of states in \( M \). [5 marks]

(d) Is the language

\[ \{a^n b^\ell a^k \in \{a, b\}^* | k \geq n + \ell \} \]

regular? Justify your answer. [5 marks]