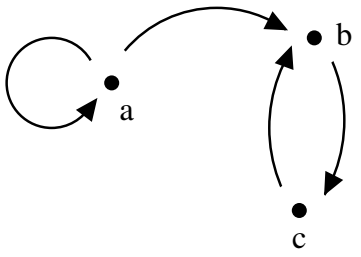
2011 Paper 1 Question 4

Discrete Mathematics I

Consider the following directed graph.



(*a*) Write down a set of ordered pairs that describes the graph. [3 marks]

(*b*) Consider the following four formulae about a relation R.

(*i*) $\forall x.\,(x,x) \in R$

(*ii*) $\forall x.\forall y.\forall z.\Big(\big((x,y) \in R \ \wedge \ (y,z) \in R\big) \ \Rightarrow \ (x,z) \in R\Big)$

(*iii*) $\forall x.\forall y.\Big((x,y) \in R \ \Rightarrow \ \exists z.\big((x,z) \in R \ \wedge \ (z,y) \in R\big)\Big)$

(*iv*) $\forall x.\exists y.\Big((x,y) \in R \ \Rightarrow \ \forall z.(x,z) \in R\Big)$

For each of the formulae,

- provide an explanation in English;
- state whether the formula holds of the relation in part (*a*) (when the domain of x, y, and z is the set $\{\mathrm{a},\mathrm{b},\mathrm{c}\}$);
- if the formula does not hold, exhibit a relation over $\{\mathrm{a},\mathrm{b},\mathrm{c}\}$ for which the formula does hold.

[14 marks]

(*c*) Write down the introduction and elimination rules for the universal quantifier in structured proof. [3 marks]