Logic and Proof

(a) Use the sequent or tableau calculus to prove the formula

$$\exists x (P(x) \to Q) \to \forall x (P(x) \to Q)$$

[6 marks]

(b) A mysterious propositional connective, $\odot$, has the following sequent calculus rule, $(\odot r)$:

$$\frac{\Gamma \Rightarrow \Delta, A, B \quad \Gamma, A, B \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, A \odot B}$$

What is the corresponding left-side sequent calculus rule, $(\odot l)$? Justify your answer, for example by giving the truth table for $\odot$.  

[6 marks]

(c) Use the DPLL method to find a model of the following set of clauses, or alternatively to prove that they are inconsistent.

$$\{P, R, \neg S\} \{\neg Q, R\} \{\neg P, \neg S, \neg R\} \{\neg P, S, Q\} \{S, Q, P\} \{\neg Q, \neg R\} \{\neg S, \neg R, P\}$$

[8 marks]