2010 Paper 6 Question 6

Logic and Proof

(a) Use the sequent or tableau calculus to prove the formula

$$\exists x\,(P(x) \to Q) \to \forall x\,(P(x) \to Q)$$

[6 marks]

(b) A mysterious propositional connective, $\odot$, has the following sequent calculus rule, ($\odot r$):

$$\frac{\Gamma \Rightarrow \Delta, A, B \qquad \Gamma, A, B \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, A \odot B}$$

What is the corresponding left-side sequent calculus rule, ($\odot l$)? Justify your answer, for example by giving the truth table for $\odot$. [6 marks]

(c) Use the DPLL method to find a model of the following set of clauses, or alternatively to prove that they are inconsistent.

$\{P, R, \neg S\}\ \{\neg Q, R\}\ \{\neg P, \neg S, \neg R\}\ \{\neg P, S, Q\}\ \{S, Q, P\}\ \{\neg Q, \neg R\}\ \{\neg S, \neg R, P\}$

[8 marks]