Algorithms II

(a) Describe Dijkstra’s shortest-path algorithm, making the priority queue operations explicit. [4 marks]

(b) Provide a small example demonstrating that Dijkstra’s shortest-path algorithm will not work correctly when negative weights are used on some arcs. [3 marks]

(c) Suppose some arcs in a directed graph have negative weights, and that \(-W\) is the least negative weight among all arcs. Suppose that we add \(W\) to all arcs in the graph to obtain a new graph with non-negative arc weights. Will the resulting graph have the same shortest paths as the original graph? Explain your answer. [3 marks]

(d) For each of the data structures listed below, describe the computational complexity of Dijkstra’s shortest-path algorithm when this data structure is used to implement the algorithm’s priority queue. Justify your answers.

(i) An unsorted array, indexed by node number. [2 marks]

(ii) A linked list, sorted by key (in this case a distance estimate). [2 marks]

(iii) A binary heap. [2 marks]

(iv) A binomial heap. [2 marks]

(v) A Fibonacci heap. [2 marks]