Probability

(a) A coin that comes up “heads” with probability $p$ is tossed $n$ times independently.

(i) What is the likelihood that $k$ of these $n$ tosses will be “heads”, and the remainder “tails”? [2 marks]

(ii) Give the mean and the variance expected for the number of “heads” outcomes. [1 mark each]

(b) In a different experiment with this same coin, you monitor how many tosses are needed before getting the first outcome of a “head”.

(i) What is the likelihood that the first “head” occurs on the $k^{th}$ trial? [2 marks]

(ii) What is the mean trial number $k$ for the first “head”, and what is the variance for this number? [1 mark each]

(c) In a Poisson process with hazard parameter $\lambda$:

(i) What is the likelihood of observing $k$ events? [2 marks]

(ii) What is the mean, and what is the variance, expected for the number of observed events? [1 mark each]

(d) If $X$ and $Y$ are random variables having expectations $E(X)$ and $E(Y)$ respectively:

(i) What is their covariance $Cov(X,Y)$? [2 marks]

(ii) In terms of their covariance $Cov(X,Y)$, and their respective variances $Var(X)$ and $Var(Y)$, what is their correlation coefficient $\rho(X,Y)$? [2 marks]

(e) For a continuous random variable $X$ that is exponentially distributed, having density function $f(x) = \lambda \exp(-\lambda x)$ for $x > 0$ and $f(x) = 0$ for $x \leq 0$:

(i) Derive the expectation $E(X)$ of this random variable. [2 marks]

(ii) Derive its variance $Var(X)$. [2 marks]