Discrete Mathematics II

The set $\Omega$ is characterised as the least set such that

if $X$ is finite and $X \subseteq \Omega$, then $X \cup \bigcup X \in \Omega$

[Recall $\bigcup X = \{ y \mid \exists x \in X. y \in x \}$.]

\( (a) \) Show

(i) $\emptyset \in \Omega$, and \hspace{2cm} [2 marks]

(ii) if $x \in \Omega$ then $\{x\} \cup x \in \Omega$. \hspace{2cm} [3 marks]

\( (b) \) State the rule(s) and the principle of rule induction appropriate for the set $\Omega$. \hspace{2cm} [3 marks]

\( (c) \) Define a set $x$ to be transitive iff

$\forall z, y. z \in y \& y \in x \Rightarrow z \in x$

Prove that all elements of $\Omega$ are transitive. \hspace{2cm} [8 marks]

\( (d) \) Describe, without proof, the elements of the set $\Omega$. \hspace{2cm} [4 marks]