Types

(a) What is meant by beta-reduction, beta-conversion and beta-normal forms for the polymorphic lambda calculus (PLC)? Explain why typeable PLC expressions are beta-convertible to beta-normal forms that are unique up to alpha-conversion. Is the same true for untypeable PLC expressions? (Any general properties of PLC you use should be clearly stated, but need not be proved.) [10 marks]

(b) Let \( \tau \) be the PLC type \( \forall \beta((\alpha \rightarrow \beta) \rightarrow \beta) \), where \( \alpha \) and \( \beta \) are distinct type variables. Give closed PLC beta-normal forms \( I \) and \( J \) with the following properties:

(i) \( I \) has type \( \forall \alpha(\alpha \rightarrow \tau) \)

(ii) \( J \) has type \( \forall \alpha(\tau \rightarrow \alpha) \)

(iii) \( \Lambda \alpha(\lambda x : \alpha(J \alpha(I \alpha x))) \) has beta-normal form \( \Lambda \alpha(\lambda x : \alpha(x)) \)

Justify your answers by giving proofs of typing and beta-conversion. [8 marks]

What is the beta-normal form of \( \Lambda \alpha(\lambda y : \tau(I \alpha (J \alpha y))) \)? [2 marks]