

2009 Paper 9 Question 15

Types

- (a) What is meant by *beta-reduction*, *beta-conversion* and *beta-normal forms* for the polymorphic lambda calculus (PLC)? Explain why typeable PLC expressions are beta-convertible to beta-normal forms that are unique up to alpha-conversion. Is the same true for untypeable PLC expressions? (Any general properties of PLC you use should be clearly stated, but need not be proved.) [10 marks]
- (b) Let τ be the PLC type $\forall\beta((\alpha \rightarrow \beta) \rightarrow \beta)$, where α and β are distinct type variables. Give closed PLC beta-normal forms I and J with the following properties:
- (i) I has type $\forall\alpha(\alpha \rightarrow \tau)$
 - (ii) J has type $\forall\alpha(\tau \rightarrow \alpha)$
 - (iii) $\Lambda\alpha(\lambda x : \alpha(J \alpha (I \alpha x)))$ has beta-normal form $\Lambda\alpha(\lambda x : \alpha(x))$

Justify your answers by giving proofs of typing and beta-conversion.

[8 marks]

What is the beta-normal form of $\Lambda\alpha(\lambda y : \tau(I \alpha (J \alpha y)))$?

[2 marks]