

## 2009 Paper 6 Question 9

### Semantics of Programming Languages

Consider the following syntax for a pure untyped functional language.

Booleans  $b \in \mathbb{B} = \{\mathbf{true}, \mathbf{false}\}$

Integers  $n \in \mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$

Variables  $x \in \mathbb{X}$  for a set  $\mathbb{X} = \{x, y, z, \dots\}$

Operations  $op ::= + \mid \geq$

Expressions

$$e ::= \mathbf{skip} \mid n \mid b \mid e_1 \ op \ e_2 \mid \mathbf{if} \ e_1 \ \mathbf{then} \ e_2 \ \mathbf{else} \ e_3 \mid \mathbf{fn} \ x \Rightarrow e \mid e_1 \ e_2 \mid x \mid \mathbf{fix} \ e$$

The language supports recursion with a fixed-point operator  $\mathbf{fix} \ e$ , which has semantics defined by the rule below.

$$\overline{\mathbf{fix} \ e} \longrightarrow e(\overline{\mathbf{fix} \ e})$$

- (a) Give the semantic rules for function application for call-by-value, call-by-name, and full-beta reduction for this language (do not give the rules for binary operators, conditional, or fix). You should define a small-step reduction relation  $e \longrightarrow e'$ , stating precisely what notion of values  $v$  you are using. [10 marks]
- (b) For the call-by-value semantics, characterise the expressions  $e$  from the grammar above that have an *immediate* runtime error in their outermost (top-level) construct. [3 marks]
- (c) For each pair of semantics (call-by-value and call-by-name, call-by-name and full-beta, and full-beta and call-by-value), give an expression with different possible termination behaviours in each element of the pair. [4 marks]
- (d) For each of your three semantics, explain a disadvantage in using that semantics for a programming language. [3 marks]