Computation Theory

(a) What is meant by a state (or configuration) of a register machine? [2 marks]

(b) A register machine program $Prog$ is said to loop at $x \in \mathbb{N}$ if, when started with register $R1$ containing $x$ and all other registers set to zero, the sequence of states $Prog$ computes contains the same non-halted state at two different times.

(i) At which $x$ does the following program loop?

\[ R0^- \rightarrow HALT \]

[2 marks]

(ii) Show that if $Prog$ loops at $x$, then the computation of $Prog$ does not halt when started with register $R1$ containing $x$ and all other registers set to zero. Is the converse true? [4 marks]

(iii) Consider the set $S = \{\langle e, x \rangle \mid Prog_e \text{ loops at } x \}$ of codes of pairs of numbers $(e, x)$ such that the register machine program $Prog_e$ with index $e$ loops at $x$. By adapting the usual proof of undecidability of the halting problem, or otherwise, show that $S$ is an undecidable set of numbers. [Hint: if $M$ were a register machine that decided membership of $S$, first consider replacing each $HALT$ instruction (and each jump to a label with no instruction) with the program in part (i).] [12 marks]