Mathematical Methods for Computer Science

(a) Consider the Markov chain, $X_n$, on the states $i = 0, 1, 2, \ldots$ with transition matrix

\[
\begin{align*}
    p_{i,i-1} &= p & i &= 1, 2, \ldots \\
    p_{i,i+1} &= 1 - p & i &= 0, 1, \ldots \\
    p_{0,0} &= p
\end{align*}
\]

where $0 < p < 1$.

(i) Show that the Markov chain is irreducible. [2 marks]

(ii) Show that the Markov chain is aperiodic. [2 marks]

(iii) Find a condition on $p$ to make the Markov chain positive recurrent and find the stationary distribution in this case. [6 marks]

(b) Consider the PageRank algorithm.

(i) Describe PageRank as a Markov chain model for motion between nodes in a graph where the nodes correspond with web pages. [5 marks]

(ii) Explain the main mathematical results that underpin the relevance of PageRank to a notion of web page “importance”. [5 marks]