2009 Paper 2 Question 9

Regular Languages and Finite Automata

Let L be a language over an alphabet Σ. The equivalence relation $\sim_L$ on the set Σ^* of finite strings over Σ is defined by $u \sim_L v$ if and only if for all $w \in \Sigma^*$ it is the case that $uw \in L$ if and only if $vw \in L$.

(a) Suppose that $L = L(M)$ is the language accepted by a deterministic finite automaton M. For each $u \in \Sigma^*$, let $s(u)$ be the unique state of M reached from the initial state after inputting the string u. Show that $s(u) = s(v)$ implies $u \sim_L v$. Deduce that for this L the number of $\sim_L$-equivalence classes is finite. [Hint: if M has n states, show that no collection of equivalence classes can contain more than n distinct elements.] [10 marks]

(b) Suppose that $\Sigma = \{a, b\}$ and L is the language determined by the regular expression $a^*b(a|b)$. Using part (a), or otherwise, give an upper bound for the number of $\sim_L$-equivalence classes for this L. [5 marks]

(c) Suppose that $\Sigma = \{a, b\}$ and $L = \{a^n b^n \mid n \geq 0\}$. By considering a^n for $n \geq 0$, or otherwise, show that for this L there are infinitely many different $\sim_L$-equivalence classes. [5 marks]