2009 Paper 2 Question 8

Probability

(a) Consider a random variable, X, taking non-negative integer values.

(i) Define the *probability generating function*, $G_X(z)$, of the random variable X. [2 marks]

(ii) Derive the expression for the expectation, $\mathbb{E}(X)$, in terms of the first derivative of $G_X(z)$. [2 marks]

(b) Calculate $G_X(z)$ in the following two cases.

(i) Suppose that X takes values equally likely from the set $\{0, 1, 2, 3, 4, 5\}$. [2 marks]

(ii) Suppose that X has the Binomial distribution $Bin(n, p)$ where $0 \leq p \leq 1$ and n a positive integer. [2 marks]

(c) Suppose that X and Y are two independent random variables each taking non-negative integer values and let their probability generating functions be $G_X(z)$ and $G_Y(z)$, respectively. Show that $X + Y$ has a probability generating function, $G_{X+Y}(z)$, given by

$$G_{X+Y}(z) = G_X(z)G_Y(z)\,.$$

[4 marks]

(d) Suppose that X and Y are independent random variables with the marginal distributions $Bin(n_1, p_1)$ and $Bin(n_2, p_2)$, respectively.

(i) Find the generating function $G_{X+Y}(z)$ and the expectation, $\mathbb{E}(X + Y)$. [4 marks]

(ii) Under what conditions on the parameters n_1, p_1 and n_2, p_2 is $X + Y$ again a Binomial distribution? [4 marks]