

2009 Paper 2 Question 5

Discrete Mathematics II

The set S of strings over symbols a and b is defined to be the least set S of strings such that

$$a \in S ,$$

$$as \in S \text{ if } s \in S , \text{ and}$$

$$bst \in S \text{ if } s \in S \text{ and } t \in S .$$

- (a) The set S may also be described as the least subset of strings closed under certain rules. Describe the rules. Write down a principle of rule induction appropriate for the set S . [5 marks]
- (b) Exhibit a derivation, indicating which rules are used, to show that the string $aabbaaa$ is in S . [4 marks]
- (c) For a string s , let $N_a(s)$ denote the number of occurrences of a in s , and similarly, let $N_b(s)$ denote the number of occurrences of b . Prove for every string $s \in S$ that $N_a(s) > N_b(s)$, i.e. there are strictly more occurrences of a than occurrences of b . [5 marks]
- (d) Exhibit a string that has strictly more occurrences of a than occurrences of b and yet is not in S . Prove that your example string is not in S . [6 marks]