Programming Methods and Java

(a) Consider the method `sum` defined in Java as

```java
public byte sum (int n) {
    byte result=0;
    for (int i=0;i<n;i++) result+=(1<<i);
    return result;
}
```

(i) What are the most positive and most negative values that can be represented by a `byte`? [2 marks]

(ii) Assuming `byte` and `int` are of arbitrary size, write down an expression for the result of `sum` in terms of `n`. For what range of `n` does `sum` actually compute the result of your expression? [4 marks]

(b) The UML class diagram below defines the software structure for an online forum, where users can post messages beneath specific topics. Topics are unique and displayed alphabetically, with associated messages displayed chronologically by creation date.

(i) Identify appropriate data structures from the Java standard library for the variables `topics` (the set of all topics) and `messages` (the set of all messages in a given topic). [4 marks]

(ii) Based on your choices, write Java code for the methods `addTopic` and `displayMessages`. [4 marks]

(iii) Using a design pattern, modify the UML diagram to allow users to "subscribe" to a topic (i.e. be notified using the method `email` when new messages are posted in that topic). Users should also be able to unsubscribe. You should identify the pattern and annotate any important methods. [6 marks]