Discrete Mathematics I

(a) State the structured-proof rules for implication introduction and disjunction elimination. [3 marks]

(b) Give either a structured proof or a counterexample for each of the following.

(i) \(((P \Rightarrow Q) \vee (P \Rightarrow R)) \Rightarrow (P \Rightarrow (Q \vee R))\)

(ii) \(((P \land Q) \Rightarrow R) \Rightarrow ((P \Rightarrow R) \land (Q \Rightarrow R))\) [8 marks]

For a set of sets \(A\), write \(\bigcup A\) for the set \(\{x \mid \exists X \in A. x \in X\}\). For a non-empty set of sets \(A\), write \(\bigcap A\) for the set \(\{x \mid \forall X \in A. x \in X\}\).

(c) Suppose \(A \subseteq \mathcal{P}(X)\) and \(B \subseteq \mathcal{P}(X)\). Prove or give a counterexample for each of the following.

(i) If \(\bigcup A\) and \(\bigcup B\) are disjoint, then \(A\) and \(B\) are disjoint.

(ii) If \(A\) and \(B\) are disjoint then \(\bigcup A\) and \(\bigcup B\) are disjoint.

(iii) If \(A\) and \(B\) are non-empty and \(\forall X \in A. \forall Y \in B. X \subseteq Y\) then \(\bigcup A \subseteq \bigcap B\). [9 marks]