Types

(a) Give an account of the Curry–Howard correspondence between the polymorphic lambda calculus (PLC) and the second-order intuitionistic propositional calculus (2IPC). Illustrate your answer by giving a proof in 2IPC of
\[ \{\} \vdash \forall p, q, r ((p \to r) \to (q \to r) \to (p \lor q) \to r) \]
corresponding to the closed PLC expression
\[ \Lambda p, q, r (\lambda x : p \to r, y : q \to r, z : p \lor q (z \ r \ x \ y)). \]
Here \( p \lor q \) is an abbreviation for \( \forall r ((p \to r) \to (q \to r) \to r) \). \[15 \text{ marks}\]

(b) Explain how \( \beta \)-reduction on PLC expressions can be used to simplify proofs in 2IPC. \[5 \text{ marks}\]