Computer Systems Modelling

(a) Suppose that you conduct a simulation experiment to estimate the mean $\mu$ of some random variable $X$. Supposing that your simulation experiment yields a sample of size $n$ of independent and identically distributed values $X_i$ derive a $100(1 - \alpha)$ percent confidence interval for the parameter $\mu$. [6 marks]

(b) Explain how you can use your confidence interval derived in part (a) to construct a rule for determining the length of your simulation so as to ensure a given size of confidence interval for the parameter $\mu$. [4 marks]

(c) Now suppose that in your simulation you can also observe a second random variable $Y$, say, with known mean value $\mu_Y$. Show that

$$E(X + c(Y - \mu_Y)) = \mu$$

where $c$ is any constant value. [4 marks]

(d) Using $Y$ as a control variate for $X$, determine the best choice of $c$ to minimise the variance of $Z = X + c(Y - \mu_Y)$. [6 marks]