2008 Paper 8 Question 13

Types

(*a*) Explain what is meant by the relation of *specialisation*, $\sigma \succ \tau$, between Mini-ML type schemes σ and Mini-ML types τ. How is $\succ$ used in the Mini-ML type system? [4 marks]

Assuming α_1 and α_2 are distinct type variables, which of the following are valid instances of specialisation?

(*i*) $\forall \alpha_1, \alpha_2 (\alpha_1 \to \alpha_2) \succ (\alpha_1 \to \alpha_1) \to \alpha_1$

(*ii*) $\forall \alpha_1 (\alpha_1 \to \alpha_2) \succ (\alpha_1 \to \alpha_1) \to \alpha_1$

(*iii*) $\forall \alpha_1 (\alpha_1 \to \alpha_2) \succ (\alpha_2 \to \alpha_2) \to \alpha_2$

(*iv*) $\forall \alpha_1 (\alpha_1 \to \alpha_1) \succ (\alpha_1 \to \alpha_1) \to \alpha_2$

[6 marks]

(*b*) Extending Mini-ML with fixed-point expressions $\mathtt{fix}\, x(M)$, consider the following typing rules:

$$\text{(mono-fix)} \quad \frac{\Gamma, x : \forall \{\}(\tau) \vdash M : \tau}{\Gamma \vdash \mathtt{fix}\, x(M) : \tau} \quad \text{if } x \notin \mathit{dom}(\Gamma)$$

$$\text{(poly-fix)} \quad \frac{\Gamma, x : \forall A(\tau) \vdash M : \tau}{\Gamma \vdash \mathtt{fix}\, x(M) : \tau} \quad \text{if } x \notin \mathit{dom}(\Gamma) \text{ and } A = \mathit{ftv}(\tau) - \mathit{ftv}(\Gamma)$$

(where as usual $\mathit{ftv}(-)$ indicates the set of free type variables in $-$). Write $\Gamma \vdash_{\text{mono}} M : \tau$ (respectively $\Gamma \vdash_{\text{poly}} M : \tau$) if $\Gamma \vdash M : \tau$ is provable in the Mini-ML type system extended with the rule (mono-fix) (respectively with the rule (poly-fix)). Let $M = \mathtt{fix}\, x(\lambda y((x\, x) y))$. State, with justification, which of the following hold for some type τ.

(*i*) $\{\} \vdash_{\text{mono}} M : \tau$

(*ii*) $\{\} \vdash_{\text{poly}} M : \tau$

[10 marks]