

2008 Paper 8 Question 12

Computer Systems Modelling

- (a) Explain what is meant by a birth–death model with birth rates λ_i and death rates μ_i in states $i = 0, 1, \dots$. You should include in your explanation the necessary probabilistic assumptions. [4 marks]
- (b) Write down the *detailed balance* equations for an equilibrium distribution, p_i , of being in state i in the birth–death model. Use these equations to determine the p_i and clarify when such an equilibrium distribution exists. [4 marks]
- (c) Consider the $M/M/m/m$ model of a loss system with m servers. Describe how this system can be used to model the behaviour of a telephone link consisting of C circuits with an arrival rate of λ calls per second and a mean holding time of $\frac{1}{\mu}$ seconds. [4 marks]
- (d) Use your general results from part (b) to derive the equilibrium distribution of the number of free circuits on a telephone link and hence deduce *Erlang's formula* for the probability that there are no free circuits available. What are the conditions for the equilibrium distribution to exist? [4 marks]
- (e) Comment on any numerical problems that could arise in calculating Erlang's formula when C is large. How might you overcome these difficulties? [4 marks]