2008 Paper 5 Question 11

Semantics of Programming Languages

Below is a fragment of L_1, equipped with a rather strange semantics. Call it $L_?$.

Booleans $b \in \mathbb{B} = \{\textsf{true}, \textsf{false}\}$
Integers $n \in \mathbb{Z} = \{\ldots, -1, 0, 1, \ldots\}$
Locations $\ell \in \mathbb{L} = \{l_0, l_1, l_2, \ldots\}$
Stores s, finite partial functions from $\mathbb{L}$ to $\mathbb{Z}$
Operations $op ::= + \mid \geq$
Expressions $e ::= b \mid n \mid !\ell \mid \ell := e \mid e_1 \;\; op \;\; e_2 \mid \textsf{if} \;\; e \;\; \textsf{then} \;\; e_1 \;\; \textsf{else} \;\; e_2$

(op +) $\langle n_1 + n_2, s\rangle \longrightarrow \langle n, s\rangle \quad$ if $n = n_1 + n_2$

(op $\geq$) $\langle n_1 \geq n_2, s\rangle \longrightarrow \langle b, s\rangle \quad$ if $b = (n_1 \geq n_2)$

(opA) $\dfrac{\langle e_1, s\rangle \longrightarrow \langle e_1', s'\rangle}{\langle e_1 \;\; op \;\; e_2, s\rangle \longrightarrow \langle e_1' \;\; op \;\; e_2, s'\rangle}$ (opB) $\dfrac{\langle e_2, s\rangle \longrightarrow \langle e_2', s'\rangle}{\langle e_1 \;\; op \;\; e_2, s\rangle \longrightarrow \langle e_1 \;\; op \;\; e_2', s'\rangle}$

(deref) $\langle !\ell, s\rangle \longrightarrow \langle n, s\rangle \quad$ if $\ell \in \mathrm{dom}(s)$ and $s(\ell) = n$

(assignA) $\langle \ell := n, s\rangle \longrightarrow \langle n, s + \{\ell \mapsto n\}\rangle \quad$ if $\ell \in \mathrm{dom}(s)$

(assignB) $\dfrac{\langle e, s\rangle \longrightarrow \langle e', s'\rangle}{\langle \ell := e, s\rangle \longrightarrow \langle \ell := e', s'\rangle}$

(ifA) $\dfrac{\langle e, s_1\rangle \longrightarrow^* \langle \textsf{true}, s_2\rangle}{\langle \textsf{if} \;\; e \;\; \textsf{then} \;\; e_1 \;\; \textsf{else} \;\; e_2, s_1\rangle \longrightarrow \langle e_1, s_1\rangle}$

(ifB) $\dfrac{\langle e, s_1\rangle \longrightarrow^* \langle \textsf{false}, s_2\rangle}{\langle \textsf{if} \;\; e \;\; \textsf{then} \;\; e_1 \;\; \textsf{else} \;\; e_2, s_1\rangle \longrightarrow \langle e_2, s_1\rangle}$

Here $\longrightarrow^*$ is the reflexive transitive closure of $\longrightarrow$, defined by:

(incl) $\dfrac{\langle e, s\rangle \longrightarrow \langle e', s'\rangle}{\langle e, s\rangle \longrightarrow^* \langle e', s'\rangle}$ (tran) $\dfrac{\langle e, s\rangle \longrightarrow^* \langle e', s'\rangle \longrightarrow^* \langle e'', s''\rangle}{\langle e, s\rangle \longrightarrow^* \langle e'', s''\rangle}$

(refl) $\dfrac{}{\langle e, s\rangle \longrightarrow^* \langle e, s\rangle}$

(*a*) Give a terminating sequence of reduction steps, with full derivations for each, of the configuration

$$\langle \textsf{if} \;\; (\ell_0 := 3) \geq 2 \;\; \textsf{then} \;\; 7 \;\; \textsf{else} \;\; 8, \{\ell_0 \mapsto 0\}\rangle \qquad \text{[5 marks]}$$

(*b*) Describe, with examples and alternative reduction rules, how the behaviour of $L_?$ expressions differs from that in L_1: for (*i*) binary operations, (*ii*) store operations, and (*iii*) conditionals. Discuss the effects that these differences could have on programming in the language. [15 marks]