Computation Theory

(a) The Halting Problem for register machines is unsolvable. State, without proof, a precise form of this result. [3 marks]

(b) Let the computation by program $c$ on data $d$ be represented by the natural number $k$ that codes the pair $(c,d)$. By considering the set $H(k)$ of the HALTing computations represented by codes $k' < k$, show that there is an increasing total function $h(k)$ which grows too fast to be computable. [6 marks]

(c) Given $h : \mathbb{N} \to \mathbb{N}$ with the above property

let $f(k) = h(k) + k$

and $g(x) = \sup\{k : f(k) \leq x\}$.

Then $f : \mathbb{N} \to \mathbb{N}$ is strictly increasing, and $g : \mathbb{N} \to \mathbb{N}$ satisfies

$g(f(k)) = k, \quad g(x) < k \quad \text{for all } x < f(k)$.

Show that $g$ grows too slowly to be computable in the following sense:

given $G : \mathbb{N} \to \mathbb{N}$ such that

(i) $\{G(n) : n \in \mathbb{N}\}$ is unbounded

(ii) $G(n) \leq g(n)$ for all $n \in \mathbb{N}$

then $G(n)$ is not computable. [11 marks]