2008 Paper 3 Question 5

Mathematical Methods for Computer Science

(a) Define the Fourier transform, $\mathcal{F}_{[f(x)]}(w)$, of a function $f(x)$ and the inverse transform to construct $f(x)$ in terms of $\mathcal{F}_{[f(x)]}(w)$.   [2 marks]

(b) Show that

$$\frac{d}{dw}\left(\mathcal{F}_{[f(x)]}(w)\right) = \mathcal{F}_{[-ixf(x)]}(w).$$

[4 marks]

(c) Define the *convolution* $f(x) * g(x)$ of two functions $f(x)$ and $g(x)$. State and prove the *convolution theorem*.   [4 marks]

(d) Consider the function $f_a(x)$ in the case where a is a positive constant defined by $f_a(x) = e^{-ax}$ for $x \geq 0$ and zero for $x < 0$. Derive the Fourier transform of $f_a(x)$.   [4 marks]

(e) Use the convolution theorem to determine the convolution $f_a(x) * f_b(x)$ where a and b are positive constants when

   (i) $a \neq b$ [3 marks]

   (ii) $a = b$.   [3 marks]

[Note: You may assume that any appropriate integrals exist and that the order of integration and differentiation may be interchanged as necessary.]