Mathematics for Computation Theory

(a) Let $f : A \rightarrow B$ be a function with domain $A$ and range $B$. Show that the relation $R$ on $A$ defined by

$$(x, y) \in R \iff f(x) = f(y)$$

is an equivalence relation. [4 marks]

(b) A partition of a set $A$ is a set $A$ of disjoint subsets of $A$ such that $A = \bigcup A$, and

$$B, C \in A \Rightarrow (B = C) \lor (B \cap C) = \emptyset$$

Let $g(n, r)$ be the number of partitions of a set $A$ having $n$ elements into $r$ subsets, where $1 \leq r \leq n$. If $1 < r < n$, show that

$$g(n, r) = r g(n-1, r) + g(n-1, r-1).$$

[7 marks]

(c) Using the above formula, or otherwise, evaluate $g(n, r)$ in the cases:

(i) $r = 2$ [4 marks]

(ii) $r = (n - 1)$ [5 marks]