COMPUTER SCIENCE TRIPOS Part II

Thursday 5 June 2008 1.30 to 4.30

PAPER 9

Answer ***five*** *questions.*

Submit the answers in five ***separate*** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* ***all*** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
None

1 Bioinformatics

(*a*) A long DNA sequence is used as a training set for parameter estimation of the DNA statistical model. The observed counts of sixteen dinucleotides N_{XY} are as follows:

$$\begin{pmatrix} & T & C & A & G \\ T & 306 & 228 & 126 & 114 \\ C & 144 & 102 & 216 & 138 \\ A & 222 & 120 & 132 & 126 \\ G & 114 & 102 & 132 & 132 \end{pmatrix}$$

Calculate:

(*i*) the transition probabilities P_{TT} and P_{AG} of the first-order Markov model of the DNA sequence; [3 marks]

(*ii*) the transition probabilities P_{TT} and P_{AG} of the first-order Markov model of the DNA sequence complementary to the given sequence. [3 marks]

(*b*) Build the tree from the following distance matrix between species A, B, C, D using the UPGMA (Unweighted Pair Group Method using arithmetic Averages) method. [7 marks]

	A	B	C	D
A		0.26	0.34	0.29
B			0.42	0.44
C				0.44
D				

(*c*) Describe how you would build a hidden Markov model (HMM) to predict protein secondary structure. [7 marks]

2 VLSI Design

(*a*) A full adder for a single bit has three inputs, a, b and c_{in}, and two outputs, s and c_{out} for the sum and carry-out. State the formulae for s and c_{out} in disjunctive normal form. [2 marks]

(*b*) Explain the operation of the following three approaches for handling carry in n-bit word adders, deriving formulae for the signals involved and explaining the limiting factors on their speed:

(*i*) ripple carry, [2 marks]

(*ii*) carry-skip with fixed-size blocks, and [6 marks]

(*iii*) carry-skip with variable-size blocks. [4 marks]

(*c*) Assuming a delay of τ for a round of combinational logic consisting of negation, conjunction and disjunction, estimate the delays for the three designs applied to a 48-bit adder. [3×2 marks]

3 Digital Communication II

(*a*) In the context of Quality of Service (QoS) in networking, what do we mean by *elastic* and *inelastic* applications? [2 marks]

(*b*) Give *two* examples of each type of application and discuss the application attributes and QoS requirements. [8 marks]

(*c*) Discuss the problems faced by traditional Internet routers in dealing with inelastic traffic. [6 marks]

(*d*) How do the IntServ and DiffServ architectures differ in offering QoS, and how might they be employed together to provide an end-to-end QoS? [4 marks]

4 Quantum Computing

(*a*) The *no-cloning theorem* is a statement that is often said to show that a quantum state $|\phi\rangle$ cannot be exactly duplicated.

(*i*) Give a mathematically precise statement of the no-cloning theorem. [2 marks]

(*ii*) Give a proof of the no-cloning theorem. [4 marks]

(*b*) The *quantum teleportation* protocol is a means by which one party, Alice, can send a quantum state to another party, Bob, by transmitting just two classical bits, provided that the two already share an entangled 2-qubit state.

Explain how the quantum teleportation protocol works, sketching any circuit that may be used. [6 marks]

(*c*) The *Deutsch–Jozsa* problem assumes that we are given a function $f : \{0, 1\} \to \{0, 1\}$ in the form of a quantum black box performing a unitary operation $U_f : |ab\rangle \mapsto |a(b \oplus f(a))\rangle$.

Sketch a circuit with only one use of U_f that determines whether f is constant or balanced. Explain carefully what measurement is performed and why it gives the desired result. [8 marks]

5 Artificial Intelligence II

A friend of mine likes to climb on the roofs of Cambridge. To make a good start to the coming week, he climbs on a Sunday with probability 0.98. Being concerned for his own safety, he is less likely to climb today if he climbed yesterday, so

$$\Pr(\texttt{climb\_today}|\texttt{climb\_yesterday}) = 0.4$$

If he did not climb yesterday then he is very unlikely to climb today, so

$$\Pr(\texttt{climb\_today}|\neg\texttt{climb\_yesterday}) = 0.1$$

Unfortunately, he is not a very good climber, and is quite likely to injure himself if he goes climbing, so

$$\Pr(\texttt{injury}|\texttt{climb\_today}) = 0.8$$

whereas

$$\Pr(\texttt{injury}|\neg\texttt{climb\_today}) = 0.1$$

(*a*) Explain how my friend's behaviour can be formulated as a *Hidden Markov Model*. What assumptions are required? [4 marks]

(*b*) You learn that on Monday and Tuesday evening he obtains an injury, but on Wednesday evening he does not. Use the *filtering* algorithm to compute the probability that he climbed on Wednesday. [8 marks]

(*c*) Over the course of the week, you also learn that he does not obtain an injury on Thursday or Friday. Use the *smoothing* algorithm to compute the probability that he climbed on Thursday. [8 marks]

6 Computer Vision

(*a*) Consider the following 2D filter function $f(x, y)$ incorporating the Laplacian operator that is often used in computer vision:

$$f(x, y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) e^{-\left(x^2+y^2\right)/\sigma^2}$$

(*i*) In 2D Fourier terms, what type of filter is this? (For example, is it a lowpass, a highpass, or a bandpass filter?) [2 marks]

(*ii*) Are different orientations of image structure treated differently by this filter, and if so, how? Which term better describes this filter: *isotropic*, or *anisotropic*? [3 marks]

(*iii*) Approximately what is the spatial frequency bandwidth of this filter, in octaves? [Hint: the answer is independent of σ.] [1 mark]

(*iv*) What is meant by image operations "at a certain scale of analysis"? In this context, define a scale-space fingerprint, and explain the role of the scale parameter. [3 marks]

(*b*) What surface properties can cause a human face to form either a Lambertian image or a specular image, or an image lying anywhere on a continuum between those two extremes? In terms of geometry and angles, what defines these two extremes of image formation? What difficulties do these factors create for efforts to extract facial structure from facial images using "shape-from-shading" inference techniques? [3 marks]

(*c*) Discuss the significance of the fact that mammalian visual systems send perhaps ten times as many corticofugal neural fibres back down from the visual cortex to the thalamus, as there are ascending neural fibres bringing visual data from the retina up to the thalamus. Does this massive neural feedback projection support the thesis of "vision as graphics", and if so how? [4 marks]

(*d*) Explain why inferring object surface properties from image properties is, in general, an ill-posed problem. In the case of inferring the colours of objects from images of the objects, how does knowledge of the properties of the illuminant affect the status of the problem and its solubility? [4 marks]

7 Advanced Graphics

(*a*) Explain how to simulate a physical system consisting of point masses and springs using the Euler method. [5 marks]

(*b*) Explain the problems exhibited by the Euler method and describe an alternative that gives a better solution. [5 marks]

(*c*) State and explain the radiosity equation. [4 marks]

(*d*) Compare and contrast the following ways of solving the radiosity equation: matrix inversion, gathering, shooting. [6 marks]

8 Specification and Verification II

(*a*) Explain how the structure of circuits can be represented in logic. Discuss the role of quantifiers and of higher-order functions and relations. [8 marks]

(*b*) Show how a predicate characterising the set of reachable states of a transition relation can be defined in higher-order logic. [4 marks]

(*c*) Write down formulae in both Computation Tree Logic (CTL) and Linear Temporal Logic (LTL) that are true whenever a property P holds of all reachable states. Define the semantics of any temporal operators that you use. [8 marks]

9 Natural Language Processing

The following shows a simple context-free grammar (CFG):

```
S -> NP VP          NP -> Kim        NP -> Sandy
NP -> Det N         N -> pudding
N -> N S            Det -> the
VP -> V             V -> likes       V -> stinks     V -> believes
VP -> V NP
VP -> V S
```

(*a*) Give a parse tree (or trees) to show the analysis this grammar assigns to the following:

the pudding Kim believes Sandy likes stinks

[3 marks]

(*b*) What is meant by the term *overgeneration*? Why might overgeneration be a problem when using a grammar for parsing grammatical text? [3 marks]

(*c*) The grammar shown above does not allow for different subcategorization properties of verbs. Assume that the following sentences should all be ungrammatical:

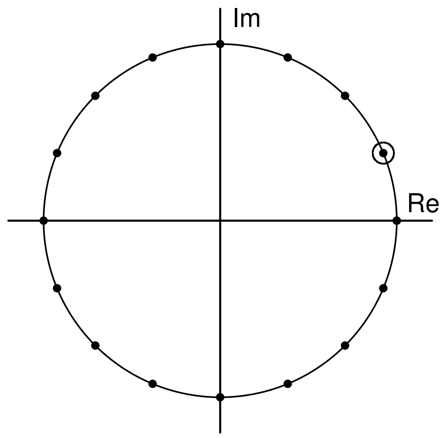
Kim likes
Kim stinks the pudding
the pudding Kim stinks stinks
the pudding Kim likes Sandy stinks

Assuming the simple CFG formalism, show how the grammar above could be modified to exclude such sentences while still admitting strings such as the one in part (*a*). [8 marks]

(*d*) Outline how subcategorization may be treated in a grammar that uses a feature structure formalism. [6 marks]

10 Information Theory and Coding

(*a*) Fast Fourier Transform algorithms use factorisation of discrete complex exponentials to avoid repeated multiplications by common factors. The diagram on the right shows a unit circle in the complex plane. The unit circle represents a continuous complex exponential (one orbit around it spans one cycle), and the 16 dots represent discrete samples of this Fourier component which need to be multiplied by 16 data points and summed to compute one discrete Fourier coefficient.



(*i*) The circled dot $\boxed{e^{2\pi i/n}}$ is a *primitive* n^{th}-root of unity, where for this diagram $n = 16$. Write down a similar expression for the full set of the n^{th}-roots of unity (all the dots), indexed by k where $1 \le k \le n$. [2 marks]

(*ii*) The 16 frequency components needed to compute the discrete Fourier transform of 16 data points are obtained by undersampling the dots; *e.g.* the 2^{nd} frequency uses every 2^{nd} dot and orbits twice. Explain the redundancy that occurs when multiplying these discrete complex exponentials by the data points. [5 marks]

(*iii*) For n data points, roughly how many multiplications are needed in a Fast Fourier Transform algorithm that avoids these redundancies? [2 marks]

(*b*) Explain the meaning of "self-Fourier," and cite at least *two* examples of mathematical objects having this property. [3 marks]

(*c*) Consider an alphabet of 8 symbols whose probabilities are as follows:

A	B	C	D	E	F	G	H
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

(*i*) If someone has selected one of these symbols and you need to discover which symbol it is by asking "yes/no" questions that will be truthfully answered, what would be the most efficient sequence of such questions that you could ask in order to discover the selected symbol? [2 marks]

(*ii*) By what principle can you claim that each of your proposed questions is maximally informative? [2 marks]

(*iii*) On average, how many such questions will need to be asked before the selected symbol is discovered? [2 marks]

(*iv*) What is the entropy of the above symbol set? [2 marks]

11 Digital Signal Processing

(*a*) A radio system outputs signals with frequency components only in the range 2.5 MHz to 3.5 MHz. The analog-to-digital converter that you want to use to digitise such signals can be operated at sampling frequencies that are an integer multiple of 1 MHz. What is the *lowest* sampling frequency that you can use without destroying information through aliasing? [5 marks]

(*b*) Consider a digital filter with an impulse response for which the z-transform is

$$H(z) = \frac{(z+1)^2}{(z-0.7-0.7\mathrm{j})(z-0.7+0.7\mathrm{j})}$$

(*i*) Draw the location of zeros and poles of this function in relation to the complex unit circle. [2 marks]

(*ii*) If this filter is operated at a sampling frequency of 48 kHz, which (approximate) input frequency will experience the lowest attenuation? [2 marks]

(*iii*) Draw a direct form I block-diagram representation of this digital filter. [5 marks]

(*c*) Make the following statements correct by changing one word or number in each case. (Negating the sentence is not sufficient.)

(*i*) Statistical independence implies negative covariance.

(*ii*) Group 3 MH fax code uses a form of arithmetic coding.

(*iii*) Steven's law states that rational scales follow a logarithmic law.

(*iv*) The Karhunen–Loève transform is commonly approximated by the z-transform.

(*v*) 40 dB corresponds to an 80× increase in voltage.

(*vi*) The human ear has about 480 critical bands.

[6 marks]

12 Computer Systems Modelling

(*a*) Suppose that you conduct a simulation experiment to estimate the mean μ of some random variable X. Supposing that your simulation experiment yields a sample of size n of independent and identically distributed values X_i derive a $100(1-\alpha)$ percent confidence interval for the parameter μ. [6 marks]

(*b*) Explain how you can use your confidence interval derived in part (*a*) to construct a rule for determining the length of your simulation so as to ensure a given size of confidence interval for the parameter μ. [4 marks]

(*c*) Now suppose that in your simulation you can also observe a second random variable Y, say, with *known* mean value μ_Y. Show that

$$E(X + c(Y - \mu_Y)) = \mu$$

where c is any constant value. [4 marks]

(*d*) Using Y as a *control variate* for X, determine the best choice of c to minimise the variance of $Z = X + c(Y - \mu_Y)$. [6 marks]

13 Types

(*a*) Give an account of the *Curry–Howard correspondence* between the polymorphic lambda calculus (PLC) and the second-order intuitionistic propositional calculus (2IPC). Illustrate your answer by giving a proof in 2IPC of

$$\{\} \vdash \forall p, q, r((p \to r) \to (q \to r) \to (p \vee q) \to r)$$

corresponding to the closed PLC expression

$$\Lambda p, q, r(\lambda x : p \to r,\ y : q \to r,\ z : p \vee q\,(z\,r\,x\,y)).$$

Here $p \vee q$ is an abbreviation for $\forall r((p \to r) \to (q \to r) \to r)$. [15 marks]

(*b*) Explain how β-reduction on PLC expressions can be used to simplify proofs in 2IPC. [5 marks]

14 Denotational Semantics

(*a*) Show that every continuous function $f : D \to D$ on a domain D has a least prefixed point, $\mathit{fix}(f)$. [3 marks]

(*b*) Let $h : P \to P$ be a continuous function on a domain P. Show that $\mathit{fix}(h) = \mathit{fix}(h \circ h)$. [3 marks]

(*c*) Let D be a domain. Let $f : D \to D$ and $g : D \to D$ be continuous functions. Define the continuous function $h : D \times D \to D \times D$ by

$$h(x, y) = (g(y), f(x))$$

for $x, y \in D$. Show

$$\mathit{fix}(h) = (\mathit{fix}(g \circ f), \mathit{fix}(f \circ g))$$

[4 marks]

(*d*) Define what it means for two terms of PCF to be *contextually equivalent*. Exhibit two terms of PCF that are contextually equivalent yet have distinct denotations in the domain $(\mathbb{B}_\perp \to (\mathbb{B}_\perp \to \mathbb{B}_\perp)) \to \mathbb{B}_\perp$ where $\mathbb{B} = \{\mathit{true}, \mathit{false}\}$ is the set of truth values. Exhibit the domain element on which the denotations differ. [10 marks]

15 Topics in Concurrency

This question assumes familiarity with the higher-order process language HOPLA, which has prefix types, function types, sum types and recursive types. Subject to suitable typings, HOPLA has transitions $t \xrightarrow{p} t'$ between closed terms t, t' and action p given by the following rules:

$$\frac{t[rec\,x\,t/x] \xrightarrow{p} t'}{rec\,x\,t \xrightarrow{p} t'} \qquad \frac{t_j \xrightarrow{p} t'}{\Sigma_{i\in I} t_i \xrightarrow{p} t'}\ j \in I$$

$$\frac{}{\mathbf{.}t \xrightarrow{\mathbf{.}} t} \qquad \frac{u \xrightarrow{\mathbf{.}} u' \quad t[u'/x] \xrightarrow{p} t'}{[u > \mathbf{.}x \Rightarrow t] \xrightarrow{p} t'}$$

$$\frac{t[u/x] \xrightarrow{p} t'}{\lambda x\, t \xrightarrow{u \mapsto p} t'} \qquad \frac{t \xrightarrow{u \mapsto p} t'}{t\,u \xrightarrow{p} t'} \qquad \frac{t \xrightarrow{p} t'}{a\,t \xrightarrow{a\,p} t'} \qquad \frac{t \xrightarrow{a\,p} t'}{\pi_a(t) \xrightarrow{p} t'}$$

(*a*) Let t be a term of type $\mathbb{P}$ with one free variable y of type $\mathbb{Q}$. Say t is *linear in* y iff for any sum of closed terms $\Sigma_{i\in I} u_i$ of type $\mathbb{Q}$

$$t[\Sigma_{i\in I} u_i/y] \sim \Sigma_{i\in I} t[u_i/y]\ .$$

(The relation $\sim$ is the bisimilarity congruence of HOPLA.)

Show from the transition semantics that the terms

$$\pi_a(y) \text{ and } [y > \mathbf{.}x \Rightarrow u] \text{ where } y \text{ is not free in } u,$$

assumed well-typed and to have only y as free variable, are linear in y. [4 marks]

(*b*) For u of sum type, let $[u > a\mathbf{.}x \Rightarrow t]$ abbreviate $[\pi_a(u) > \mathbf{.}x \Rightarrow t]$.

Why is $[y > a\mathbf{.}x \Rightarrow t]$ linear in y, where y is not free in t? [2 marks]

Derive a rule for the transitions of $[u > a\mathbf{.}x \Rightarrow t]$. [2 marks]

Show $[a\mathbf{.}u > a\mathbf{.}x \Rightarrow t] \sim t[u/x]$ and $[b\mathbf{.}u > a\mathbf{.}x \Rightarrow t] \sim nil$ if $b \neq a$. (The term *nil* represents the empty sum.) [4 marks]

(*c*) Describe the type you would use to interpret CCS in HOPLA. [2 marks]

Write down a HOPLA term that realises the parallel composition of CCS. What is its type? [2 marks]

State the expansion law for CCS parallel composition. In a few sentences, indicate how, using part (*b*), you would derive the expansion law from your definition of parallel composition. [4 marks]

16 Optimising Compilers

Consider the language

$$e ::= x \mid \lambda x.e \mid e_1 e_2 \mid e_1; e_2 \mid \xi?x.e \mid \xi!e_1.e_2 \mid \texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3.$$

in which ξ represents a communication channel (from a fixed set), and the forms $e_1; e_2$, $\xi?x.e$ and $\xi!e_1.e_2$ respectively represent sequencing, reading from a channel (binding x) and writing to a channel.

(*a*) Construct an *effect system* for the above language where effects, F, are represented as *sets* of actions of the form $\xi?$ or $\xi!$ representing side-effects of input from or output to ξ. Explain the two principal occurrences of effects in the judgement form of your system. [8 marks]

(*b*) Assess the *safety* of your analysis making clear any respects in which execution behaviour may fail to match your analysis. [2 marks]

(*c*) Let us say a general program analysis framework is *any-path*-like (as opposed to *all-path*-like) if the analyses of $\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3$ and $e_1; e_2; e_3$ coincide. Is your effect system any-path-like? Justify your answer. [2 marks]

(*d*) Augment the above language with constructs

$$e ::= \texttt{letchan}\ \xi\ \texttt{in}\ e \ \mid\ \texttt{parsum}(e_1, e_2)$$

which allow a *local channel* to be created, and also inter-thread communication (e_1 and e_2 are evaluated in parallel and their sum returned when both have completed). Extend your effect system to the augmented language, noting that reads and writes to local channels are *not* to be reflected in the overall effect of a `letchan`. [6 marks]

(*e*) Suggest an alternative data structure for F that might enable effects of the form "after getting two inputs from channel ξ_1 *or* getting one input from channel ξ_2 then an output is written to channel ζ" to be represented. [A modified effect system is not required.] [2 marks]

END OF PAPER