COMPUTER SCIENCE TRIPOS Part IA

Tuesday 3 June 2008 1.30 to 4.30

PAPER 2

Answer **one** *question from each of Sections A, B and C, and* **two** *questions from Section D.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

STATIONERY REQUIREMENTS
Script paper
Blue cover sheets
Tags

SPECIAL REQUIREMENTS
None

SECTION A

1 Digital Electronics

(*a*) Briefly explain the differences between *combinational* and *sequential* logic. [2 marks]

(*b*) With the aid of appropriate diagrams, briefly explain the operation of Moore and Mealy finite state machines and highlight their differences. [6 marks]

(*c*) The state sequence for a binary counter is as follows:

A	B	C	D
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0

The counter is to be implemented using four synchronously clocked D-type flip-flops.

(*i*) Draw a state table for the counter, showing the required D inputs. [4 marks]

(*ii*) Find expressions for the D inputs, making use of unused states if appropriate. [6 marks]

(*iii*) What problem could occur when the counter circuit is powered-up? Give *two* possible general methods for overcoming the problem. [2 marks]

2 Digital Electronics

(*a*) With the aid of relevant diagrams, show the effect on the output of a combinational logic circuit of a:

(*i*) static hazard;

(*ii*) dynamic hazard. [3 marks]

(*b*) Simplify the following expressions using Boolean algebra:

(*i*) $X = (A + \overline{B} + \overline{A} \,.\, B) \,.\, (A + \overline{B}) \,.\, \overline{A} \,.\, B$

(*ii*) $Y = (A + \overline{B} + \overline{A} \,.\, B) \,.\, \overline{C}$ [4 marks]

(*c*) Given:

$$F = A \,.\, B \,.\, \overline{C} \,.\, D + A \,.\, C + B \,.\, \overline{C} \,.\, \overline{D} + \overline{B} \,.\, C + \overline{A} \,.\, \overline{C} \,.\, \overline{D} + \overline{A} \,.\, \overline{B} \,.\, \overline{C} \,.\, D$$

(*i*) Show using a Karnaugh map that F can be simplified to

$$F_1 = A \,.\, B + \overline{A} \,.\, \overline{B} + A \,.\, C + B \,.\, \overline{C} \,.\, \overline{D}$$

[2 marks]

(*ii*) Show that there are a total of four possible expressions for F. [3 marks]

(*iii*) Show how F_1 can be implemented using NAND gates and draw the circuit diagram. Assume that complemented input variables are available. [2 marks]

(*iv*) Show how the static 1 hazard in F_1 can be eliminated using a Karnaugh-map-based approach. [2 marks]

(*v*) Now implement F_1 assuming that only 2-input NAND gates are available. [4 marks]

SECTION B

3 Discrete Mathematics

Let X and Y be sets. You are reminded that a relation from X to Y is a subset of the product $X \times Y$.

(a) Explain what it means for a relation f from X to Y to be a *function*, an *injection* and a *surjection* from X to Y. [4 marks]

(b) A *bijection* from X to Y is defined to be a function from X to Y which is both an injection and a surjection. Prove that a function f from X to Y is a bijection iff it has an inverse function g, i.e. g is a function from Y to X such that $g \circ f = id_X$ and $f \circ g = id_Y$.
[Remember to prove both the "if" and "only if" parts of the assertion.]
[12 marks]

(c) Describe, without proof, a bijection from $\mathcal{P}(X \times Y)$ to $(X \to \mathcal{P}(Y))$ and its inverse. [4 marks]

4 Discrete Mathematics

Let I be a non-empty subset of the natural numbers $\mathbb{N} = \{1, 2, 3, \cdots\}$.

The set S is defined to be least subset of $\mathbb{N}$ such that

> $I \subseteq S$, and
> if $m, n \in S$ and $m < n$, then $(n - m) \in S$.

Define h to be the least member of S. This question guides you through to a proof that h coincides with the *highest common factor* of I, written $hcf(I)$, and defined to be the natural number with the properties that

> $hcf(I)$ divides n for every element $n \in I$, and
> if k is a natural number which divides n for every $n \in I$, then k divides $hcf(I)$.

[Throughout this question you may assume elementary facts about division.]

(*a*) The set S may also be described as the least subset of $\mathbb{N}$ closed under certain rules. Describe the rules. Write down a principle of rule induction appropriate for the set S. [4 marks]

(*b*) Show by rule induction that $hcf(I)$ divides n for every $n \in S$. [3 marks]

(*c*) Let $n \in S$. Establish that

$$\text{if } \ p.h < n \ \text{ then } \ (n - p.h) \in S$$

for all non-negative integers p. [5 marks]

(*d*) Show that h divides n for every $n \in S$. [Hint: suppose otherwise and derive a contradiction.] [5 marks]

(*e*) Explain very briefly why the results of parts (*b*) and (*d*) imply that $h = hcf(I)$. [3 marks]

SECTION C

5 Probability

(*a*) Suppose that X is a random variable whose value r is distributed Geometric(p). Write down the expression for the probability $\mathrm{P}(X = r)$. [3 marks]

(*b*) By using a suitable generating function or otherwise, show that the expectation $\mathrm{E}(X) = (1 - p)/p$. [5 marks]

The University Computing Service define a serious power outage as a power cut that lasts for longer than their Uninterruptable Power Supply equipment can maintain power. During the course of an academical year the number of serious power outages is a random variable whose value is distributed Geometric(2/5). Accordingly, the probability of having no serious power outages during the course of a year is 2/5.

(*c*) The University is investigating a compensation scheme which would make no payment over the year if the number of serious power outages were zero or one but which would pay the Computing Service £1000 for every such outage (including the first) if the total number of serious power outages in a year were two or more. Determine the expected annual sum that the Computing Service would receive. [8 marks]

(*d*) To what value would the parameter of the Geometric Distribution have to be changed (from 2/5) for the expected annual sum to be £750? [4 marks]

6 Probability

(*a*) Give a brief account of the Trinomial Distribution and include in your explanation an expression that is equivalent to $\frac{n!}{r!(n-r)!}\, p^r q^{n-r}$ for the Binomial Distribution. [5 marks]

(*b*) An indicator light can be in one of three states: OFF, FLASHING and ON, with probabilities 1/2, 2/5 and 1/10 respectively. A test panel has five such lights whose states are mutually independent.

(*i*) What is the probability that all five lights are OFF? [3 marks]

(*ii*) What is the probability that three lights are OFF, one light is FLASHING and one light is ON? [3 marks]

(*iii*) What is the probability that three or more lights are OFF and at most one is ON? [9 marks]

All results *must* be expressed as fractions.

SECTION D

7 Software Design

Software systems often incorporate structural representations of the application domain in which they operate. For example, a vehicle control system should be aware of the fact that the car has precisely four wheels. This kind of information must be *captured*, *encoded* and *tested* at each stage of the software design process. Using the number of wheels in a car as a simple example, describe relevant design activities and products at *each* of the following phases of a software project:

(*a*) inception; [4 marks]

(*b*) elaboration; [4 marks]

(*c*) construction; [4 marks]

(*d*) transition; [4 marks]

(*e*) system operation. [4 marks]

8 Regular Languages and Finite Automata

(*a*) Explain what is a *context-free grammar* and the language it generates. [4 marks]

(*b*) What does it mean for a context-free grammar to be regular? Given any deterministic finite automaton M, describe a regular context-free grammar that generates the language of strings accepted by M. [4 marks]

(*c*) Construct a non-deterministic finite automaton with ε-transitions whose language of accepted strings is equal to the language over the alphabet $\{a, b, c\}$ generated by the context-free grammar with non-terminals q_0 and q_1, whose start symbol is q_0 and whose productions are $q_0 \to abq_1$, $q_1 \to \varepsilon$, $q_1 \to q_0$ and $q_1 \to abc$. [4 marks]

(*d*) Is every language generated by a context-free grammar equal to the set of strings accepted by some non-deterministic finite automaton with ε-transitions? Justify your answer. (Any standard results about regular languages you use should be carefully stated, but need not be proved.) [8 marks]

9 Professional Practice and Ethics

(*a*) The British Computer Society Code of Conduct has four sections. What kind of professional conduct does each section cover, and how does each of these kinds of conduct benefit the profession and its members? [8 marks]

(*b*) True or False questions:

(*i*) A User can provide occasional use of the University computer system for a friend who is a temporary visitor.

(*ii*) Circumstances that mitigate minor infractions of the rules promulgated by the Information Technology Syndicate include, among other things, inebriation.

(*iii*) Appropriate use of the Cambridge University Data Network (CUDN) means *bona fide* academic activity plus a low level for private purposes.

(*iv*) Small amounts of commercial activity are acceptable as long as the User is acting in a private capacity.

[4 marks]

(*c*) The IT industry is increasingly aware of its own environmental impact. Describe at least *one* environmental problem to which the industry contributes and how, as an IT professional, you can help to solve this problem. [4 marks]

(*d*) "Social engineering is a greater threat to computer security than computer cracking software." What is social engineering and what measures can be taken to guard against it? [2 marks]

(*e*) What is *copyleft* and how is it used to protect free, open-source, software? [2 marks]

END OF PAPER