

## 2007 Paper 9 Question 14

### Types

- (a) Give the rules inductively defining the type system of the *polymorphic lambda calculus* (PLC). [5 marks]
- (b) What does it mean for a PLC expression  $M$  to be in *beta-normal form*? [2 marks]
- (c) The *long normal forms*,  $L$ , and the *neutral forms*,  $N$ , are special kinds of PLC expression given by the following grammar:

$$\begin{aligned} L &::= \lambda x : \tau(L) \mid \Lambda \alpha(L) \mid N, \\ N &::= x \mid N L \mid N \tau. \end{aligned}$$

- (i) Arguing by induction on the structure of  $L$  and  $N$ , or otherwise, show that all such expressions are in beta-normal form. [4 marks]
- (ii) Show that if  $N$  is a neutral form, then  $\{\} \vdash N : \tau$  is not provable in the PLC type system for any type  $\tau$ , where  $\{\}$  is the empty typing environment. (You may assume without proof that if  $\Gamma \vdash M : \tau$  is provable in the PLC type system, then the free variables of the expression  $M$  are contained in the domain of definition of the typing environment  $\Gamma$ .) [3 marks]
- (iii) Hence prove that for any long normal form  $L$ ,  $\{\} \vdash L : \forall \alpha(\alpha)$  is not provable in the PLC type system. [6 marks]