

## 2007 Paper 9 Question 12

### Quantum Computing

- (a) Consider a quantum finite automaton with two basis states,  $|0\rangle$  being the start state and  $|1\rangle$  the only accepting state. The automaton operates on a two-letter alphabet, with matrices  $M_a = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$  and  $M_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ .

What are the probabilities that the automaton accepts each of the following input strings?

- (i)  $a$  [3 marks]
- (ii)  $aba$  [3 marks]
- (iii)  $abb$  [3 marks]
- (b) Give a complete description of the probabilities of acceptance associated with various possible input strings. [5 marks]
- (c) Prove that there is no *two-state* probabilistic automaton with the same behaviour as the automaton described in part (a). [6 marks]