(a) Define the notion of a functional dependency. [2 marks]

(b) Consider the following “rule” for functional dependencies.

\[ A \rightarrow B \text{ and } B, C \rightarrow D, \text{ then } A, C \rightarrow D. \]

Either prove this rule is correct, or present a counter-example showing that the rule is false. [4 marks]

(c) The union rule for functional dependencies states that if \( F \models X \rightarrow Y \) and \( F \models X \rightarrow Z \), then \( F \models X \rightarrow Y \cup Z \) (this can also be written as \( F \models X \rightarrow Y, Z \)).

Prove this rule using only Armstrong’s axioms. [4 marks]

(d) Suppose that \( R(A, B, C) \) is a relational schema. Write a relational algebra query that evaluates to the empty set exactly when the functional dependency \( B \rightarrow C \) holds on \( R \). [4 marks]

(e) The schema \( R(A, B, C, D, E) \) has the following functional dependencies.

\[
\begin{align*}
A &\rightarrow B, C \\
C, D &\rightarrow E \\
B &\rightarrow D \\
E &\rightarrow A
\end{align*}
\]

Is \( D, E \) a candidate key for \( R \)? Explain your answer. [6 marks]