Computation Theory

(a) What does it mean for a set of natural numbers $S \subseteq \mathbb{N}$ to be

(i) *recursive*? [1 mark]

(ii) *recursively enumerable*? [2 marks]

(b) Show that if a set is recursive, then it is also recursively enumerable. [5 marks]

(c) Let $\phi_e$ denote the partial function from $\mathbb{N}$ to $\mathbb{N}$ computed by the register machine with code $e \in \mathbb{N}$. Is either of the following sets of numbers recursively enumerable? Justify your answer in each case, stating clearly any standard results that you use.

(i) $S_1 = \{ e \in \mathbb{N} \mid \text{for all } x \in \mathbb{N}, \phi_e(x) \text{ is defined} \}$. [6 marks]

(ii) $S_2 = \{ e \in \mathbb{N} \mid \text{for some } x \in \mathbb{N}, \phi_e(x) \text{ is defined} \}$. [6 marks]