Complexity Theory

(a) Give a precise definition of polynomial-time reductions. [2 marks]

(b) Give a precise definition of NP-completeness. [3 marks]

(c) Let **Subset Sum** denote the following decision problem:

Given a set of positive integers \( S = \{v_1, \ldots, v_n\} \) and a number \( t \), determine whether there is a subset of \( S \) that sums to exactly \( t \).

(i) Explain why **Subset Sum** is in NP. [3 marks]

(ii) Describe a polynomial-time reduction from the problem of 3-dimensional matching to **Subset Sum**. [9 marks]

(iii) Explain why parts (i) and (ii) above imply that **Subset Sum** is NP-complete. [3 marks]