Computation Theory

(a) (i) Define the notion of a register machine and the computations that it carries out. [5 marks]

(ii) Explain, in general terms, what is meant by a universal register machine. (You should make clear what scheme for coding programs as numbers you are using, but you are not required to describe a universal register machine program in detail.) [5 marks]

(b) (i) Explain what it means for a partial function \( f \) from \( \mathbb{N} \) to \( \mathbb{N} \) to be computable by a register machine. [2 marks]

(ii) Let \( n > 1 \) be a fixed natural number. Show that the partial function from \( \mathbb{N} \) to \( \mathbb{N} \)

\[
f_n(x) = \begin{cases} nx & \text{if } x > 0 \\ \text{undefined} & \text{if } x = 0 \end{cases}
\]

is computable. [3 marks]

(iii) Explain why there are only countably many computable functions from \( \mathbb{N} \) to \( \mathbb{N} \). Deduce that there exists a partial function from \( \mathbb{N} \) to \( \mathbb{N} \) that is not computable. (Any standard results you use about countable and uncountable sets should be clearly stated, but need not be proved.) [3 marks]

(iv) If a partial function \( f \) from \( \mathbb{N} \) to \( \mathbb{N} \) is computable, how many different register machine programs are there that compute \( f \)? [2 marks]