Digital Signal Processing

(a) Consider a software routine that converts and records the audio samples received in a digital telephone network call (8 kHz sampling frequency, 8 bit/sample) into a WAV file (8 kHz sampling frequency, 16 bit/sample, uniform quantisation). Your colleague attempted to write a very simple conversion routine for this task, but the resulting audio is very distorted.

(i) Name two variants of the method used for quantising the amplitude of audio samples in digital telephone networks and explain one of them.

(ii) Your colleague’s routine right-pads each 8-bit data word from the telephone network with eight additional least-significant zero bits to obtain 16-bit values. Explain how this distorts the signal by discussing which frequencies could appear at the output when the incoming telephone signal consists of a pure 1 kHz sine tone.

(b) A real-valued discrete random sequence \( \{x_i\} \) is fed into a linear time-invariant filter with impulse response \( h_0 = 1, h_3 = 1 \), and \( h_i = 0 \) for all other \( i \). We observe for the resulting output sequence \( \{y_i\} \) the expected value

\[
\mathbb{E}(y_{i+k} \cdot x_i) = \begin{cases} 
1 & \text{for } k = -1 \\
2 & \text{for } k = 0 \\
1 & \text{for } k = 1 \\
1 & \text{for } k = 2 \\
2 & \text{for } k = 3 \\
1 & \text{for } k = 4 \\
0 & \text{otherwise}
\end{cases}
\]

What is the value of the autocorrelation sequence \( \phi_{xx}(k) \)?

(c) The \( YCrCb \) colour encoding is used in many image compression methods.

(i) How is it defined and why is it used?

(ii) Is the conversion from \( 3 \times 8 \)-bit \( RGB \) to \( 3 \times 8 \)-bit \( YCrCb \) coordinates fully reversible? Why?