2006 Paper 5 Question 9

Logic and Proof

(*a*) For each of the following equivalences, state whether it holds or not, justifying each answer rigorously.

(*i*) $((P \to Q) \to R) \to P \quad \simeq \quad P \vee (Q \wedge \neg R)$ [3 marks]

(*ii*) $(\forall x\, P(x)) \to (\exists x\, Q(x)) \quad \simeq \quad \exists x\, [P(x) \to Q(x)]$ [5 marks]

(*b*) For the following formula, present *either* a sequent calculus proof *or* a falsifying interpretation.
$$[\forall x\, \exists y\, P(x, y)] \to [\forall x\, P(x, f(x))]$$
[4 marks]

(*c*) For the following formula, in modal logic S4, present *either* a sequent calculus proof *or* a falsifying interpretation.

$$\Box(A \wedge \Diamond B) \to (\Box A \wedge \Diamond B)$$
[8 marks]