

## 2006 Paper 5 Question 8

### Databases

- (a) Define Boyce–Codd normal form. [3 marks]
- (b) Suppose that a relation  $R$  has  $n$  attributes. How many distinct functional dependencies could be defined for  $R$ ? [3 marks]
- (c) The *union rule for functional dependencies* states that if  $F \models X \rightarrow Y$  and  $F \models X \rightarrow Z$ , then  $F \models X \rightarrow Y \cup Z$  (this can also be written as  $F \models X \rightarrow Y, Z$ ).

Prove this rule using only Armstrong's axioms. [5 marks]

- (d) *Heath's Theorem* states that if  $R(A, B, C)$  satisfies the functional dependency  $A \rightarrow B$ , where  $A$ ,  $B$ , and  $C$  are disjoint non-empty sets of attributes, then

$$R = \pi_{A,B}(R) \bowtie_A \pi_{A,C}(R),$$

where  $\bowtie_A$  is the equi-join on the attributes of  $A$ . Prove this theorem.

[9 marks]