Databases

(a) Define Boyce–Codd normal form. [3 marks]

(b) Suppose that a relation $R$ has $n$ attributes. How many distinct functional dependencies could be defined for $R$? [3 marks]

(c) The union rule for functional dependencies states that if $F \models X \rightarrow Y$ and $F \models X \rightarrow Z$, then $F \models X \rightarrow Y \cup Z$ (this can also be written as $F \models X \rightarrow Y, Z$).

Prove this rule using only Armstrong’s axioms. [5 marks]

(d) Heath’s Theorem states that if $R(A, B, C)$ satisfies the functional dependency $A \rightarrow B$, where $A$, $B$, and $C$ are disjoint non-empty sets of attributes, then

$$R = \pi_{A,B}(R) \boweq_A \pi_{A,C}(R),$$

where $\boweq_A$ is the equi-join on the attributes of $A$. Prove this theorem. [9 marks]