

2006 Paper 5 Question 10

Foundations of Functional Programming

In this question you will be using a polymorphic type-reconstruction algorithm similar to that used in ML. Your answer will be expected to contain sufficient explanation of the overall method to explain how it applies in the particular cases considered.

- (a) The fixed-point operator Y satisfies the equation $Y f = f(Y f)$. Follow through your general type deduction algorithm on this equation, and *either* deduce a polymorphic type for Y *or* show that one does not exist. [5 marks]
- (b) The lambda-expression

$$\lambda f.(\lambda g.f(gg))(\lambda g.f(gg))$$

behaves as a fixed-point operator.

- (i) Show that it satisfies the equation given in part (a). [5 marks]
- (ii) Perform type-deduction on this lambda-expression and again either deduce a type for it or show that one cannot be found. [5 marks]
- (c) In the context of type-derivation, explain relationships and differences between the following two lines of ML-style code:

```
let fun I x = x in (I 1, I 2, I 3) end;  
(lambda I => (I 1, I 2, I 3)) (lambda x => x);
```

[5 marks]